Georgia End-Of-Course Tests

Content Descriptions

Based on the state-mandated content standards

Analytic Geometry

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"Making Education Work for All Georgians"

Introduction

The State Board of Education is required by Georgia law (A+ Educational Reform Act of 2000, O.C.G.A. §20-2-281) to adopt End-of-Course Tests (EOCT) designed to measure student achievement in core subjects in grades nine through twelve. With educator input and State Board of Education approval, eight content areas were designated in 2001 to be tested. The current state-mandated content standards in mathematics were adopted by the State Board of Education in July 2010, and the Analytic Geometry EOCT was developed based on these standards.

Program Purpose

The EOCT are designed to improve student achievement by assessing student performance on the standards specific to each course tested. Student performance on each test is provided to schools for diagnostic and remedial use. These results are used to help make instruction more effective and to ensure that all Georgia students have access to rigorous courses that meet high academic expectations. These results are also used for student accountability and to gauge the quality of education in the state. The EOCT are the final exams for the courses specified. For students enrolled in grade nine for the first time before July 1, 2011, the final grade for each course is calculated by weighing the course grade 85% and the EOCT score 15%. For students enrolled in grade nine for the first time on July 1, 2011, or after, the final grade for each course is calculated by weighing the course grade 80% and the EOCT score 20% (State Board Rule 160-4-2-.13). The student must have a final grade of at least 70 to pass the course and earn credit toward graduation.

EOCT Content Descriptions

The EOCT Content Descriptions are provided to acquaint Georgia educators with the content assessed by the EOCT. Only the knowledge, concepts, and skills addressed in the state-mandated content standards are assessed on the EOCT. It is important to note that some content standards are better suited for classroom or individual assessment rather than large-scale summative assessment. While those standards designed for classroom/individual assessment are not included in the Content Descriptions, the knowledge, concepts, and skills outlined are often required for the mastery of the standards that are assessed. Therefore, the EOCT Content Descriptions are in *no way* intended to substitute for the state-mandated content standards; they are provided to help educators better understand how the standards will be assessed. Further, the EOCT Content Descriptions *by no means* suggest *when* concepts and skills should be introduced in the instructional sequence; rather, they are intended only to communicate the concepts and skills that will be assessed on the EOCT, but in no particular order. Georgia law requires educators to teach the material set forth in the state-mandated content standards. The standards are located at www.georgiastandards.org.

Analytic Geometry Domains
In order to provide reliable measures of student achievement, as well as to give structure to the assessment program, the performance standards were grouped into content domains. Each domain was created by combining standards that share similar content characteristics. Four domains were identified for Analytic Geometry:

- **Geometry**
  Students will understand similarity in terms of transformations; understand congruence in terms of rigid motions; prove geometric theorems; make geometric constructions; define trigonometric ratios and use them to solve problems involving right triangles; understand and apply properties of circles, and use them in determining related measures such as arc lengths and areas of sectors; explain and use volume formulas to solve problems; translate between geometric descriptions and equations of conic sections; use coordinates to prove simple geometric theorems algebraically.

- **Expressions, Equations, and Functions**
  Students will perform arithmetic operations on polynomials; interpret the structure of expressions and write expressions in equivalent forms; create equations that describe numbers or relationships; solve equations and inequalities in one variable; solve systems of equations; interpret and analyze functions using different representations; build functions that model relationships; construct and compare linear, quadratic, and exponential function models.

- **Number and Quantity**
  Students will extend the properties of exponents to rational numbers; use properties of rational and irrational numbers; perform arithmetic operations with complex numbers; use complex numbers in polynomial identities and equations.

- **Statistics and Probability**
  Students will represent bivariate data on a scatter plot; describe relationships for bivariate data; understand conditional probability, including independence; use the rules of probability to compute probabilities of compound events.

Standards for Mathematical Practice
The state-mandated content standards in mathematics require content to be taught in conjunction with the mathematical practices identified in the *Common Core State Standards for Mathematics*. These mathematical practice standards are necessary for students to master each of the mathematics content standards. Problem solving, reasoning, representation, connections, and communication are the critical dimensions of mathematical proficiency that all students need.

The concepts and skills inherent in the practice standards are integrated in items across the four content domains.

Standards for Mathematical Practice

1. Students will make sense of problems and persevere in solving them.
2. Students will reason abstractly and quantitatively.
3. Students will construct viable arguments and critique the reasoning of others.
4. Students will model with mathematics.
5. Students will use appropriate tools strategically.
6. Students will attend to precision.
7. Students will look for and make use of structure.
8. Students will look for and express regularity in repeated reasoning.

Associated Concepts and Skills

- Building new mathematical knowledge through problem solving
- Solving problems that arise in mathematics and in other contexts
- Applying and adapting a variety of appropriate strategies to solve problems
- Monitoring and reflecting on the process of mathematical problem solving
- Recognizing reasoning and proof as fundamental aspects of mathematics
- Making and investigating mathematical conjectures
- Developing and evaluating mathematical arguments and proofs
- Selecting and using various types of reasoning and methods of proof
- Organizing and consolidating mathematical thinking through communication
- Communicating mathematical thinking coherently and clearly to peers, teachers, and others
- Analyzing and evaluating mathematical thinking and strategies of others
- Using the language of mathematics to express mathematical ideas precisely
- Recognizing and using connections among mathematical ideas
- Understanding how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognizing and applying mathematics in contexts outside of mathematics
- Creating and using representations to organize, record, and communicate mathematical ideas
- Selecting, applying, and translating among mathematical representations to solve problems
- Using representations to model and interpret physical, social, and mathematical phenomena

Analytic Geometry
Domain: Geometry

Overview of the Domain

- Students will understand similarity in terms of similarity transformations.
- Students will prove theorems involving similarity.
- Students will understand congruence in terms of rigid motion.
- Students will prove geometric theorems.
- Students will make geometric constructions.
- Students will define trigonometric ratios and solve problems involving right triangles.
- Students will understand and apply theorems about circles.
- Students will find arc lengths and areas of sectors of circles.
- Students will explain volume formulas and use them to solve problems.
- Students will translate between the geometric descriptions and equations of conic sections.
- Students will use coordinates to prove simple geometric theorems algebraically.

Associated Standards

MCC9-12.G.CO (6, 7, 8, 9, 10, 11, 12, 13)
MCC9-12.G.SRT (1, 2, 3, 4, 5, 6, 7, 8)
MCC9-12.G.C (1, 2, 3, 5)
MCC9-12.G.GPE (1, 2, 4)
MCC9-12.G.GMD (1, 3)

Associated Concepts and Skills

Assessment of this domain will focus on the student's ability to

- understand congruence in terms of rigid motion
  - predict the effect of a given rigid motion on a given figure
  - given two figures, use the definition of congruence in terms of rigid motions to decide if the figures are congruent
  - explain how the criteria for triangle congruence (ASA, SAS, SSS) follow from the definition of congruence in terms of rigid motion
- understand congruence of triangles
  - know that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent
  - explain and use the Angle-Side-Angle criteria (ASA) to show that two triangles are congruent
  - explain and use the Side-Angle-Side criteria (SAS) to show that two triangles are congruent
  - explain and use the Side-Side-Side criteria (SSS) to show that two triangles are congruent

- prove geometric theorems about lines and angles, including:
  - theorems about vertical angles
  - when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent
  - points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints
- prove geometric theorems about triangles, including:
  - a line parallel to one side of a triangle divides the other two proportionally
  - the Pythagorean Theorem, using triangle similarity
  - the measures of interior angles of a triangle have a sum of 180º
  - base angles of isosceles triangles are congruent
  - the segment joining the midpoints of two sides of a triangle is parallel to the third side and half the length
  - the medians of a triangle meet at a point
- prove geometric theorems about parallelograms, including:
  - opposite sides of a parallelogram are congruent
  - opposite angles of a parallelogram are congruent
  - diagonals of a parallelogram bisect each other
  - rectangles are parallelograms with congruent diagonals
- make formal geometric constructions
  - copy a segment and an angle
  - bisect a segment and an angle
  - construct perpendicular lines, including the perpendicular bisector of a line segment
  - construct a line parallel to a given line through a point not on the line
  - construct an equilateral triangle inscribed in a circle
  - construct a square inscribed in a circle
  - construct a regular hexagon inscribed in a circle
- verify the properties of dilations given by a center and a scale factor
  - understand that a dilation takes a line not passing through the center of the dilation to a parallel line
  - understand that a dilation leaves a line passing through the center of the dilation unchanged
  - understand that a line segment is longer under a dilation when the scale factor is greater than 1
  - understand that a line segment is shorter under a dilation when the scale factor is between 0 and 1
- determine whether two figures are similar
  - use the definition of similarity in terms of similarity transformations
  - justify similarity as the equality of all corresponding pairs of angles
  - justify similarity as the proportionality of corresponding pairs of sides
  - use the Angle-Angle criteria (AA) to show that two triangles are similar

- understand and apply the basic trigonometric ratios for right triangles
  - apply trigonometric ratios to find unknown measures of sides
  - solve real-life problems involving trigonometric ratios of right triangles
  - solve for the length of any side of a right triangle, given sufficient information
  - work from a model presented graphically or verbally, including situations involving angles of elevation and/or depression
- explain the relationship between the trigonometric ratios of complementary angles
  - understand that $\sin \theta = \cos (90° - \theta)$ and $\cos \theta = \sin (90° - \theta)$
- understand and use properties of and relationships among radii, chords, tangents, and secants of circles as an application of triangle similarity, for example,
  - relationship of tangent to radius to point of tangency
  - relationship of the segments formed in a circle by intersecting chords
  - congruence of tangents from a given point outside the circle
- understand and use properties of and relationships among angles related to circles, such as central, inscribed, circumscribed, and related angles, for example,
  - relationship between arc measures and angle measures
  - relationship between measures of central angles and inscribed angles
  - relationship between angles formed outside circles and the arc formed by the sides of the angle
- use the properties of circles to solve problems involving the length of an arc and the area of a sector
- give informal arguments for why volume formulas of cylinders, pyramids, and cones are true
- write and interpret the equation of a circle
  - derive the equation of a circle given its center and radius
  - derive the formula for a circle using the Pythagorean Theorem
  - complete the square to find the center and radius of a circle given by an equation
- recognize, write, and interpret equations of parabolas
  - derive the equation of a parabola, given its focus and directrix
- use coordinates to prove simple geometric theorems algebraically, for example,
  - prove or disprove that a figure defined by four given points is a special quadrilateral, such as a parallelogram, rectangle, square, or trapezoid
  - use the distance formula and slope formula to prove or disprove properties of geometric figures given on the coordinate plane
  - prove properties involving circles
  - prove properties involving parabolas
- understand, use, and apply circumference and area formulas
- understand, use, and apply formulas for volume
  - calculate volume of a cylinder
  - calculate volume of a pyramid
  - calculate volume of a cone
  - calculate volume of a sphere

Analytic Geometry
Domain: Expressions, Equations, and Functions

Overview of the Domain

- Students will perform arithmetic operations on polynomials.
- Students will interpret the structure of expressions.
- Students will write expressions in equivalent forms to solve problems.
- Students will create equations that describe numbers or relationships.
- Students will solve equations in one variable.
- Students will solve inequalities in one variable.
- Students will solve systems of equations.
- Students will interpret functions that arise in applications in terms of the context.
- Students will analyze functions using different representations.
- Students will build functions that model relationships between quantities.
- Students will build new functions from existing functions.
- Students will construct and compare linear, quadratic, and exponential models and solve problems.

Associated Standards

MCC9-12.A.SSE (1, 1a, 1b, 2, 3, 3a, 3b)
MCC9-12.A.APR.1
MCC9-12.A.CED (1, 2, 4)
MCC9-12.A.REI (4, 4a, 4b, 7)
MCC9-12.F.IF (4, 5, 6, 7, 7a, 8, 8a, 9)
MCC9-12.F.BF (1, 1a, 1b, 3)
MCC9-12.F.LE.3

Associated Concepts and Skills

Assessment of this domain will focus on the student's ability to

- interpret expressions that represent a quantity in terms of its context
  - interpret terms in a quadratic expression
  - interpret factors in a quadratic expression
  - interpret coefficients in a quadratic expression
  - interpret complicated expressions by viewing one or more of their parts as a single entity
- rewrite expressions as equivalent expressions
  - rearrange formulas to highlight a quantity of interest
  - write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function
  - compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions)

- factor general quadratic expressions completely over the integers
- understand that the set of polynomials is closed under the operations of addition, subtraction, and multiplication
- add, subtract, and multiply polynomials
- operate with polynomials, with an emphasis on expressions that can be rewritten in linear or quadratic forms
- write quadratic equations and inequalities and use those equations to solve problems
- graph quadratic functions
  - explain why the graph of every quadratic function is a transformation of the graph of the basic function $f(x) = x^2$
  - relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes
  - find and interpret the average rate of change of a quadratic function
  - show key features of the graph of a quadratic function such as intercepts, maxima, and minima
- solve general quadratic equations
  - use inspection to solve quadratic equations
  - take square roots to solve quadratic equations
  - use factoring to solve quadratic equations
  - apply the vertex form of a quadratic function to find real solutions of quadratic equations that cannot be solved by factoring
  - use the method of completing the square to solve quadratic equations
  - use and justify the quadratic formula
  - explore complex numbers as non-real solutions of quadratic equations
- solve applications involving quadratic equations
  - solve problems with quadratic functions that model the behavior of objects that are thrown in the air and allowed to fall subject to the force of gravity
  - solve problems with quadratic functions to reveal the maximum or minimum value of the function it defines
- solve quadratic inequalities graphically
  - use exact solutions of quadratic equations to give exact values for the endpoints of the intervals in the solutions of quadratic inequalities
- solve systems of equations
  - solve a simple system consisting of a linear equation and a quadratic equation in two variables graphically
  - solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically

Analytic Geometry
Domain: Number and Quantity

Overview of the Domain

- Students will extend the properties of exponents to rational numbers.
- Students will use properties of rational and irrational numbers.
- Students will perform arithmetic operations with complex numbers.
- Students will use complex numbers in polynomial identities and equations.

Associated Standards

MCC9-12.N.RN (1, 2, 3)
MCC9-12.N.CN (1, 2, 7)

Associated Concepts and Skills

Assessment of this domain will focus on the student's ability to

- define rational exponents
  - apply product of powers, quotient of powers, power of a power, power of a product, and power of a quotient to rewrite and/or evaluate expressions
  - understand that for any real number a, $a^0 = 1$, $a^{-n} = \frac{1}{a^n}$, and $a^{\frac{1}{n}} = \sqrt[n]{a}$, and apply these properties
- rewrite expressions involving radicals and rational exponents
- explore properties of rational and irrational numbers
  - explain why the sum of two rational numbers is rational
  - explain why the product of two rational numbers is rational
  - explain why the sum of a rational number and an irrational number is irrational
  - explain why the product of a non-zero rational number and an irrational number is irrational
- define the imaginary number i
  - understand that $i = \sqrt{-1}$ and $i^2 = -1$
- define complex numbers
- add, subtract, and multiply complex numbers
  - apply the associative, distributive, and commutative properties
- solve quadratic equations with real coefficients that have complex solutions
  - use the quadratic formula

Analytic Geometry
Domain: Statistics and Probability

Overview of the Domain

- Students will represent bivariate data on a scatter plot and describe how the variables are related.
- Students will understand independence and conditional probability.
- Students will use the rules of probability to compute probabilities of compound events.

Associated Standards

MCC9-12.S.ID (6, 6a)
MCC9-12.S.CP (1, 2, 3, 4, 5, 6, 7)

Associated Concepts and Skills

Assessment of this domain will focus on the student's ability to

- create and interpret scatter plots
  - plot bivariate data values (for quadratic models) on a scatter plot
  - describe how two quantitative variables shown on a scatter plot are related
  - fit a quadratic function to given data
  - use quadratic functions fitted to data to solve problems in the context of the data
- use elementary set theory and notation
  - use set notation as a way to algebraically represent complex networks of events or real world objects
  - identify subsets of a set
  - find the intersections of two sets
  - find the union of two sets
  - find the complements of sets
- determine whether certain events are independent
  - understand and apply concepts of conditional probability to identify if events are independent
  - understand that two events, A and B, are independent if the probability of A and B occurring together is the product of their probabilities
  - understand that two events, A and B, are independent if the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B
- construct and interpret two-way frequency tables
  - use a two-way frequency table as a sample space to decide if events are independent

- find conditional probabilities
  - understand the conditional probability of A given B as the ratio of P(A and B) to P(B)
  - use a two-way frequency table as a sample space to approximate conditional probabilities
- find probabilities of compound events
  - understand and apply the addition rule P(A or B) = P(A) + P(B) – P(A and B)
  - interpret a conditional probability answer in terms of a uniform probability model