Georgia End-Of-Course Tests

Content Descriptions

Based on the Georgia Performance Standards

Mathematics I

Algebra / Geometry / Statistics

Dr. John D. Barge, State School Superintendent

"Making Education Work for All Georgians"

Introduction

The State Board of Education is required by Georgia law (A+ Educational Reform Act of 2000, O.C.G.A. §20-2-281) to adopt End-of-Course Tests (EOCT) designed to measure student achievement in core subjects in grades nine through twelve. With educator input and State Board of Education approval, eight content areas were designated in 2001 to be tested. The Georgia Performance Standards (GPS) in mathematics were adopted by the State Board of Education in July 2004, and the Mathematics I EOCT was developed based on these standards.

Program Purpose

The EOCT are designed to improve student achievement by assessing student performance on the standards specific to each course tested. Student performance on each EOCT is provided to schools for diagnostic and remedial use. These results are used to help make instruction more effective and to ensure that all Georgia students have access to a rigorous curriculum that meets high academic standards. These results are also used for student accountability and to gauge the quality of education in the state. The EOCT are the final exams for each EOCT course. For students in grade 10 or above beginning the 2011-2012 school year, the final grade in each course is calculated by weighing the course grade 85% and the EOCT score 15%. For students in grade 9 beginning the 2011-2012 school year and later, the final grade in each course is calculated by weighing the course grade 80% and the EOCT score 20% (State Board Rule 160-4-2-.13). The student must have a final grade of at least 70 to pass the course and to earn credit toward graduation.

EOCT Content Descriptions

The EOCT Content Descriptions are provided to acquaint Georgia educators with the content coverage of the EOCT. Only the knowledge, concepts, and skills addressed in the GPS are assessed on the EOCT. Committees of Georgia educators reviewed the curriculum and provided guidance for the assessment program.

It is important to note that some curricular standards are better suited for classroom or individual assessment rather than large-scale, summative assessment. While those curricular standards designed for classroom/individual assessment are not included in the Content Descriptions, the knowledge, concepts, and skills outlined are often required for the mastery of the standards that are assessed. Therefore, the EOCT Content Descriptions are in *no way* intended to substitute for the GPS; they are provided to help educators better understand how the curriculum will be assessed. Further, the EOCT Content Descriptions *by no means* suggest *when* concepts and skills should be introduced in the instructional sequence; rather, their purpose is to communicate when concepts and skills will be assessed on the EOCT. Georgia law requires educators to teach the standards set forth in the state-adopted curriculum (i.e., the GPS). The GPS are located at www.georgiastandards.org.

Mathematics I: Algebra/Geometry/Statistics Domains
To provide reliable measures of student achievement and to give structure to the assessment program, the content standards contained in the GPS were grouped into content domains. Each domain was created by combining standards that share similar content characteristics. Three domains were identified for Mathematics I: Algebra/Geometry/Statistics.

- Algebra
  Students will demonstrate the ability to explore functions; solve radical, simple quadratic and rational equations; simplify and perform operations with radical, polynomial, and rational expressions.

- Geometry
  Students will demonstrate the ability to explore, understand, and use the formal language of reasoning and justification in both algebraic and geometric contexts; apply properties of polygons; and determine distances and points of concurrence.

- Data Analysis and Probability
  Students will demonstrate the ability to determine probability; use both permutations and combinations to find the number of outcomes; pose questions to be answered by collecting data; and organize, represent, investigate, interpret, and make inferences from data.

Process Standards

The GPS in mathematics require content to be taught in conjunction with process skills identified as the process standards. These process standards are necessary for students to master each of the mathematics content standards. Problem solving, reasoning, representation, connections, and communication are the critical dimensions of mathematical proficiency that all students need.

The concepts and skills inherent in the process standards are integrated in items across the three content domains.

Overview of the Process Standards

- Students will solve problems (using appropriate technology).
- Students will reason and evaluate mathematical arguments.
- Students will communicate mathematically.
- Students will make connections among mathematical ideas and with other disciplines.
- Students will represent mathematics in multiple ways.

Associated GPS Standards

MM1P1 through MM1P5 within content from MM1A1 through MM1D3

Associated GPS Concepts and Skills

- Building new mathematical knowledge through problem solving.
- Solving problems that arise in mathematics and in other contexts.
- Applying and adapting a variety of appropriate strategies to solve problems.
- Reflecting on and monitoring the process of mathematical problem solving.
- Recognizing reasoning and proof as fundamental aspects of mathematics.
- Making and investigating mathematical conjectures.
- Developing and evaluating mathematical arguments and proofs.
- Selecting and using various types of reasoning and methods of proof.
- Organizing and consolidating mathematical thinking through communication.
- Communicating mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyzing and evaluating mathematical thinking and strategies of others.
- Using the language of mathematics to precisely express mathematical ideas.
- Recognizing and using connections among mathematical ideas.
- Understanding how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognizing and applying mathematics in contexts outside of mathematics.
- Creating and using representations to organize, record, and communicate mathematical ideas.
- Selecting, applying, and translating mathematical representations to solve problems.
- Using representations to model and interpret physical, social, and mathematical phenomena.

Mathematics I: Algebra/Geometry/Statistics
Domain: Algebra

Overview of the Domain

- Students will use graphs, tables, and simple algebraic techniques to explore and interpret the characteristics of functions.
- Students will simplify and perform operations with radical expressions, polynomials, and rational expressions.
- Students will solve radical, simple quadratic, and rational equations.

Associated GPS Standards

MM1A1 MM1A2 MM1A3

Associated GPS Concepts and Skills

Assessment of this domain will focus on student ability to

- use function notation ($f(x)$ read as f of x) to represent functions and to convey functional relationships presented in tables, graphs, and algebraic form
- graph and identify graphs of basic functions (limited to $f(x) = x^n$, where $n = 1$ to 3, $f(x) = |x|$, $f(x) = \sqrt{x}$ and $f(x) = \frac{1}{x}$)
  - select a graph that matches a particular function
  - select a function that matches a given graph
  - understand that graphs are geometric representations of functions
- graph transformations of basic functions
  - examine and identify vertical shifts, vertical stretches, and vertical shrinks of parent functions
  - explore and identify reflections across the x- and y-axes of parent functions
- investigate and explain the characteristics of quadratic, cubic, inverse, absolute value, and square root functions (using linear functions only as a building block)
  - identify a domain (the set of inputs) and a range (the set of outputs)
  - understand set notation
  - explore the zeros/solutions
  - find x- and y-intercepts
  - determine intervals of increase and decrease
  - locate maximum and minimum values
  - explain end behavior
- relate the characteristics of a function to a given context
  - utilize graphs, tables, and words to explain and predict the behavior of a function
  - understand the distinctions between discrete and continuous domains
- recognize sequences as functions with domains that are whole numbers greater than zero
  - examine sequences given in tables, algebraically, or by producing a context and identifying the corresponding function
  - understand the difference between finite and infinite sequences
  - explore how and when to use a recursive definition for a given pattern or sequence

- explore rates of change
  - compare graphs of functions that have a constant rate of change (i.e., slope) versus graphs that have variable rates of change
  - compare rates of change of linear, quadratic, square root, and other function families
  - explore average rates of change in regard to speed, cost, revenue, and other real-world applications
- determine graphically and algebraically whether a nonlinear function has symmetry
  - identify if a function is even, odd, or neither
  - interpret if a given function has symmetry
- understand that in any equation x can be interpreted as the equation $f(x) = g(x)$
  - interpret the solutions as the x-value(s) of the intersection points of the graphs of $y_1 = f(x)$ and $y_2 = g(x)$
  - use algebra to find the value of x that makes $f(x) = g(x)$ true
  - understand that functions are equal if they have the same domain and rule of correspondence
- simplify algebraic expressions involving square roots
- perform mathematical operations with square roots
  - understand when to rationalize a denominator
  - comprehend the equivalence of a simplified square root expression and the equivalence of a nonsimplified square root expression
- add, subtract, multiply, and divide polynomials
- use the Binomial Theorem to expand binomials, limited to the third power
- add, subtract, multiply, and divide rational algebraic expressions
- factor expressions involving the difference/sum of two squares, difference/sum of two cubes, and trinomials in the form $ax^2 + bx + c = 0$, where $a = 1$ and factor methods limited to the greatest common factor, grouping, trial and error, and special products
- utilize area formulas of polygons and volume models of prisms, cylinders, etc., for polynomial arithmetic
- use either factorization or square roots to solve quadratic equations in the form $ax^2 + bx + c = 0$, where $a = 1$
- solve simple radical equations by isolating the variable and squaring both sides
- use technology, tables, and graphs to solve equations resulting from the investigation of $x^2 + bx + c = 0$
  - interpret the solution of a quadratic function from a graph of the data
  - identify and comprehend the meaning of the x-intercepts from a table of quadratic data
- solve simple rational equations that result in linear or quadratic equations with a leading coefficient of 1

Mathematics I: Algebra/Geometry/Statistics
Domain: Geometry

Overview of the Domain

- Students will investigate properties of geometric figures in a coordinate plane.
- Students will understand and use the language of mathematical argument and justification.
- Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.

Associated GPS Standards

MM1G1 MM1G2 MM1G3

Associated GPS Concepts and Skills

Assessment of this domain will focus on student ability to

- determine the distance between two points on a coordinate grid
  - find distances between two points on the same horizontal or vertical line
  - use various methods (such as the distance formula or Pythagorean theorem) to calculate the distance when given two points with coordinates (x_1, y_1) and (x_2, y_2)
- calculate the distance between a point and a line on a coordinate grid
  - understand that distance between a point and a line is measured along a perpendicular
  - explore and understand perpendicular lines
  - utilize the distance formula or other methods when appropriate
- calculate the midpoint of a segment
  - determine the midpoint of a horizontal or a vertical line
  - use various methods (such as the midpoint formula, similar triangles, averaging the endpoints, etc.) to locate the midpoint when given two points on a coordinate grid with coordinates (x_1, y_1) and (x_2, y_2)
  - find an endpoint of a line segment when given its other endpoint and midpoint
- understand the distance formula as an application of the Pythagorean theorem
  - explore how the distance formula is derived from the Pythagorean theorem
  - find the length of a hypotenuse or a leg of a triangle plotted on a coordinate grid
- use the coordinate plane to investigate properties of and verify conjectures related to triangles and quadrilaterals
  - use relationship properties of side measures, slopes, diagonals, etc., of triangles and quadrilaterals to determine unknown side lengths
  - use side and angle theorems to prove triangles and quadrilaterals are similar and/or congruent
  - understand the minimal information necessary to conclude that two triangles are congruent
  - utilize properties of parallel and perpendicular lines and angle bisectors to construct or draw the missing measure of a polygon, given a known relationship to another triangle or quadrilateral

  - utilize the distance formula to classify figures as triangles and quadrilaterals (e.g., squares, rectangles, trapezoids, kites, parallelograms, and rhombuses)
  - determine missing vertices of a triangle or a quadrilateral by utilizing side and angle relationships of a given figure
- use conjecture, inductive reasoning, deductive reasoning, counterexamples, and indirect proof, as appropriate, in mathematical and real-world applications
  - utilize prior knowledge of quadrilateral relationships to prove or disprove classification of quadrilaterals
  - utilize paragraph proofs, flow proofs, two-column proofs, or any other method that relays clear communication to justify conclusions regarding polygon relationships
- explore and use the relationships among conditional statements
  - determine the hypothesis and conclusion of a conditional statement, in word or in mathematical form
  - write the converse of a conditional statement by exchanging the hypothesis and conclusion
  - realize that the inverse of a conditional statement is the negation of the hypothesis and conclusion of the conditional statement
  - understand that the contrapositive of a conditional statement is the negation of the hypothesis and conclusion of the conditional statement and then the interchange of the hypothesis and conclusion
  - utilize conditional statements to prove algebraic, geometric, and real-world concepts
- determine the sum of interior and exterior angles in a polygon
  - utilize angle relationships of a polygon to find a missing measure or the total interior angles measures of a specific polygon
  - utilize angle relationships, such as linear pairs and the exterior angle sum theorem, to determine an exterior angle of a polygon
- understand inequality theorems involving triangles
  - apply the triangle inequality theorem to determine if given side lengths form a triangle
  - utilize the side-angle inequality theorem to determine the largest and smallest angle or side in a triangle
  - use the exterior-angle inequality theorem, linear pairs, or the sum of the angles of a triangle adding to 180° to determine the measure of an exterior angle of a triangle when given two remote interior angles
- understand congruence postulates and theorems for triangles
  - identify and use SSS, SAS, ASA, AAS, HL to prove/justify that given triangles are congruent through proofs including two-column, paragraph, and flow chart, or any other valid form of communication
  - understand that SSA and AAA are not valid methods to prove triangle congruency
- use and prove properties of and relationships among the following special quadrilaterals:
  - parallelograms—understand that the opposite sides are congruent, the opposite angles are congruent, the consecutive angles are supplementary, and the diagonals bisect each other

  - rectangles—understand that the diagonals are congruent and that rectangles have all the properties of a parallelogram
  - rhombuses—understand that the diagonals are perpendicular and bisect a pair of opposite angles and that rhombuses have all the properties of a parallelogram
  - squares—understand that the diagonals are perpendicular and congruent and that squares have all the properties of a parallelogram
  - isosceles trapezoids—understand that they have only one pair of parallel sides and congruent diagonals
  - kites—understand that two pairs of consecutive sides are congruent, the diagonals are perpendicular, one diagonal is bisected, and angles between non-congruent sides are congruent to each other
- use properties to identify and classify quadrilaterals
- use theorems to find unknown angle and side measures
- find and use points of concurrency, such as incenter, orthocenter, circumcenter, and centroid, in triangles
  - use bisectors, medians, and altitudes to find points of concurrency
  - locate centers of circles inscribed in or circumscribed about triangles
  - make decisions about which center best meets a given set of conditions

Mathematics I: Algebra/Geometry/Statistics
Domain: Data Analysis and Probability

Overview of the Domain

- Students will determine the number of outcomes related to a given event.
- Students will use the basic laws of probability.
- Students will relate samples to a population.
- Students will explore variability of data by determining the mean absolute deviation (the average of the absolute values of the deviations).

Associated GPS Standards

MM1D1 MM1D2 MM1D3 MM1D4

Associated GPS Concepts and Skills

Assessment of this domain will focus on student ability to

- apply the addition and multiplication principles of counting
- calculate and use simple permutations and combinations
  - integrate the multiplication principle to clarify the difference between permutations and combinations and when each is appropriate to use for a situation
  - use diagrams to justify the classification
  - utilize permutation and combination formulas to determine the number of possible arrangements of real-world events
- understand when an event is mutually exclusive and use diagrams, tables, and the formula $P(A \text{ or } B) = P(A) + P(B)$ to calculate the probability of mutually exclusive events
- use diagrams, tables, and the formula $P(A \text{ and } B) = P(A) \bullet P(B|A)$ to find the probabilities of dependent events and understand when an event is dependent
- use diagrams, tables, and the formula $P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$ to calculate conditional probabilities of real-world events
- use expected value to predict outcomes and make inferences
- compare summary statistics from one sample data distribution to another sample data distribution
  - interpret the mean, median, quartiles, and interquartile range of multiple data sets
  - understand normal and binomial data distributions
  - describe center and variability of data distributions
- compare the averages of summary statistics from a large number of samples to the corresponding population parameters
  - extract information from two data sets (such as the set from an overall population to the set for a sample)
  - interpret histograms to compare data sets
- understand that a random sample is used to improve the chance of selecting a representative sample
  - determine the type of sampling to be used, given a scenario, so that a survey yields results from a random population sample

  - understand that a random sample will yield unbiased results
- explore the variability of data by determining the mean absolute deviation (the average of the absolute values of the deviations) and find the greatest or least mean absolute deviation of a data set