GEORGIA’S K-12
MATHEMATICS STANDARDS
2021

Calculus

MATHEMATICS
KEY COMPETENCIES &
COURSE STANDARDS
WITH
LEARNING OBJECTIVES
IN PROGRESSION ORDER

GEORGIA'S K-12 MATHEMATICS STANDARDS 2021

Governor Kemp and Superintendent Woods are committed to the best set of academic standards for Georgia's students – laying a strong foundation of the fundamentals, ensuring age- and developmentally appropriate concepts and content, providing instructional supports to set our teachers up for success, protecting and affirming local control and flexibility regarding the use of mathematical strategies and methods, and preparing students for life. These Georgia-owned and Georgia-grown standards leverage the insight, expertise, experience, and efforts of thousands of Georgians to deliver the very best educational experience for Georgia's 1.7 million students.

In August 2019, Governor Brian Kemp and State School Superintendent Richard Woods announced the review and revision of Georgia's K-12 mathematics standards. Georgians have been engaged throughout the standards review and revision process through public surveys and working groups. In addition to educator working groups, surveys, and the Academic Review Committee, Governor Kemp announced a new way for Georgians to provide input on the standards: the Citizens Review Committee, a group composed of students, parents, business and community leaders, and concerned citizens from across the state. Together, these efforts were undertaken to ensure Georgians will have buy-in and faith in the process and product.

The Citizens Review Committee provided a charge and recommendations to the working groups of educators who came together to craft the standards, ensuring the result would be usable and friendly for parents and students in addition to educators. More than 14,000 Georgians participated in the state's public survey from July through September 2019, providing additional feedback for educators to review. The process of writing the standards involved more than 200 mathematics educators -- from beginning to veteran teachers, representing rural, suburban, and metro areas of our state.

Grade-level teams of mathematics teachers engaged in deep discussions; analyzed stakeholder feedback; reviewed every single standard, concept, and skill; and provided draft recommendations. To support fellow mathematics teachers, they also developed learning progressions to show when key concepts were introduced and how they progressed across grade levels, provided examples, and defined age/developmentally appropriate expectations.

These teachers reinforced that strategies and methods for solving mathematical problems are classroom decisions -- not state decisions -- and should be made with the best interest of the individual child in mind. These recommended revisions have been shared with the Academic Review Committee, which is composed of postsecondary partners, age/development experts, and business leaders, as well as the Citizens Review Committee, for final input and feedback.

Based on the recommendation of Superintendent Woods, the State Board of Education will vote to post the draft K-12 mathematics standards for public comment. Following public comment, the standards will be recommended for adoption, followed by a year of teacher training and professional learning prior to implementation.

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Overview

This document contains a draft of Georgia's 2021 K-12 Mathematics Standards for the High School Calculus Course, which is a fourth mathematics course option in the high school course sequence.

The standards are organized into big ideas, course competencies/standards, and learning objectives/expectations. The grade level key competencies represent the standard expectation of learning for students in each grade level. The competencies/standards are each followed by more detailed learning objectives that further explain the expectations for learning in the specific grade levels.

New instructional supports are included, such as clarification of language and expectations, as well as detailed examples. These have been provided for teaching professionals and stakeholders through the Evidence of Student Learning Column that accompanies each learning objective.

Course Description:

Calculus is a fourth-year math option for students who have completed Pre-Calculus. The course provides students with the opportunity to develop an understanding of the derivative and its applications as well as the integral and its applications. Throughout the course there should be a focus on notational fluency and the use of multiple representations. The course includes the study and analysis of limits and continuity as applied to a variety of functions; the derivative as related to limits and continuity; various derivative rules such as product, quotient, and chain; applications of the derivative including curve analysis, applied max/min situations, related rate problems, and use of Mean Value Theorem; the definite integral as a limit of Riemann sums; properties of definite integrals; the Fundamental Theorem of Calculus as it relates derivatives and integrals; techniques of integration including u-substitution; and applications of the integral including solving separable differential equations, finding a particular solution curve given an initial condition, area between curves on a coordinate plane, and average value situations.

Topics should be analyzed in multiple ways, to include verbal and written, numerical, algebraic, and graphical presentations. Instruction and assessment should include the appropriate use of technology. Concepts should be introduced and investigated, where appropriate, in the context of realistic phenomena.

Prerequisite:

This course is designed for students who have successfully completed *Pre-Calculus*.

Georgia's K-12 Mathematics Standards - 2021
Mathematics Big Ideas and Learning Progressions, High School

Mathematics Big Ideas, HS

HIGH SCHOOL
MATHEMATICAL PRACTICES (MP)
MATHEMATICAL MODELING (MM)
NUMERICAL (QUANTITATIVE) REASONING (NR)
PATTERNING & ALGEBRAIC REASONING (PAR)
FUNCTIONAL & GRAPHICAL REASONING (FGR)
GEOMETRIC & SPATIAL REASONING (GSR)
DATA & STATISTICAL REASONING (DSR)
PROBABILISTIC REASONING (PR)

The 8 Mathematical Practices and the Mathematical Modeling Framework are essential to the implementation of the content standards presented in this course. More details related to these concepts can be found in the links below and in the first two standards presented in this course:

Mathematical Practices

Mathematical Modeling Framework

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The seven course standards listed below are the key content competencies students will be expected to master in this course. Additional clarity and details are provided through the classroom-level learning objectives and evidence of student learning details for each course standard found on subsequent pages of this document.

COURSE STANDARDS
C.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.
C.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.
C.FGR.2: Apply limit notation and characteristics of continuity to analyze behaviors of functions.
C.FGR.3: Relate limits and continuity to the derivative as a rate of change and apply it to a variety of situations including modeling contexts.
C.FGR.4: Apply derivatives to situations in order to draw conclusions including curve analysis and modeling rates of change in applications.
C.GSR.5: Analyze the relationship between the derivative and the integral using the Fundamental Theorem of Calculus.
C.PAR.6: Apply the definite integral and indefinite integral to contextual situations.

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MATHEMATICAL MODELING		
C.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.		
Expectations		**Evidence of Student Learning** (not all inclusive; see Course Overview for more details)
C.MM.1.1	Explain contextual, mathematical problems using a mathematical model.	***Fundamentals*** • Students should be provided with opportunities to learn mathematics in the context of real-life problems. • Contextual, mathematical problems are mathematical problems presented in context where the context makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (model with mathematics).
C.MM.1.2	Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.	***Fundamentals*** • Students should be able to use the content learned in this course to create a mathematical model to explain real-life phenomena.
C.MM.1.3	Using abstract and quantitative reasoning, make decisions about information and data from a contextual situation.	
C.MM.1.4	Use various mathematical representations and structures with this information to represent and solve real-life problems.	

FUNCTIONAL & GRAPHICAL REASONING – *Relationship Between Limits and Continuity*		
C.FGR.2: Apply limit notation and characteristics of continuity to analyze behaviors of functions.		
Expectations		**Evidence of Student Learning** (not all inclusive; see Course Overview for more details)
Determine limits graphically, numerically, and analytically.		
C.FGR.2.1	Estimate limits from graphs and tables of values.	***Fundamentals*** • Include both two-sided and one-sided limit notation.
C.FGR.2.2	Find limits of sums, differences, products, and quotients using substitution.	
C.FGR.2.3	Represent asymptotic behavior using limits.	***Relevance and Application*** • Include vertical and horizontal asymptotes.
C.FGR.2.4	Find limits of rational functions using algebraic techniques.	
C.FGR.2.5	Demonstrate continuity at a point using the definition and limit notation.	***Fundamentals*** • Include discontinuities of point, jump, and infinite.

C.FGR.2.6	Apply the Intermediate Value Theorem to a function over a closed interval.	***Relevance and Application*** • Include the existence of roots of polynomials.

***FUNCTIONAL & GRAPHICAL REASONING** – Differentiation*		
C.FGR.3: Relate limits and continuity to the derivative as a rate of change and apply it to a variety of situations including modeling contexts.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
Apply the definition of the derivative.		
C.FGR.3.1	Interpret the derivative as an instantaneous rate of change that is a two-sided limit of an average rate of change.	
C.FGR.3.2	Demonstrate and apply the relationship between differentiability and continuity.	
C.FGR.3.3	Apply the concept of derivative geometrically, numerically, and analytically.	
Apply rules of differentiation.		
C.FGR.3.4	Find the derivatives of sums, products, quotients, and composite functions.	***Strategies and Methods*** • Blend chain rule with prior rules.
C.FGR.3.5	Find the derivatives of a variety of relations.	***Strategies and Methods*** • Include algebraic, trigonometric, inverse, logarithmic, and exponential functions as well as implicitly defined curves.
C.FGR.3.6	Calculate higher order derivatives.	***Fundamentals*** • Include the first derivative, second derivative, and general nth derivative.

***FUNCTIONAL & GRAPHICAL REASONING** – Applications of Differentiation*		
C.FGR.4: Apply derivatives to situations in order to draw conclusions including curve analysis and modeling rates of change in applications.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
Analyze function behavior using the derivative.		
C.FGR.4.1	Calculate the slope of a curve at a point.	***Example*** • Include both zero and undefined slopes.

C.FGR.4.2	Write the equation of the tangent line to a curve at a point and use it to obtain a local linear approximation of a value near the point of tangency.	
C.FGR.4.3	Identify intervals where functions are increasing, decreasing, and constant by using the relationship between the function and the sign of its first derivative.	
C.FGR.4.4	Identify points of inflection and intervals of concavity of a function by using the second derivative of a function.	
C.FGR.4.5	Compare characteristics of f, f', and f" graphically, numerically, analytically, and with technology.	***Example*** • Relate the functions in ways such as: f(x) is concave upward on an interval when f'(x) increases which also means that f"(x) is positive.
C.FGR.4.6	Apply Mean Value Theorem.	***Example*** • Examine function behavior in context graphically, in a table, and real-world applications.
Apply the derivative to real-world problems.		
C.FGR.4.7	Apply Extreme Value Theorem.	***Relevance and Application*** • Examine function behavior to determine absolute maxima and minima including real-world applications.
C.FGR.4.8	Apply the derivative to real-world problems to find both local and absolute extrema, with and without technology.	***Relevance and Application*** • Include optimization problems.
C.FGR.4.9	Model rates of change in applied situations.	***Relevance and Application*** • Include related rate problems and one-dimensional particle motion with velocity and acceleration.

***GEOMETRIC & SPATIAL REASONING** – Indefinite and Definite Integrals*		
C.GSR.5: Analyze the relationship between the derivative and the integral using the Fundamental Theorem of Calculus.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
Evaluate and interpret definite integrals.		
C.GSR.5.1	Use Riemann sums to approximate values of definite integrals.	***Strategies and Methods*** • Include left rectangle, right rectangle, midpoint rectangle, and trapezoids as well as equal and unequal subdivisions.
C.GSR.5.2	Interpret a definite integral as a limit of Riemann sums.	
C.GSR.5.3	Find the exact value of a definite integral using geometric formulas on a coordinate plane.	***Strategies and Methods*** • Include circles, triangles, rectangles, and trapezoids.

C.GSR.5.4	Demonstrate the use of properties of definite integrals.	***Fundamentals*** • $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$ $\int_a^b kf(x)dx = k\int_a^b f(x)dx$ $\int_a^a f(x)dx = 0$ $\int_a^b f(x)dx = -\int_b^a f(x)dx$ $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$ If $f(x) \leq g(x)$ on [a, b], then $\int_a^b f(x)dx \leq \int_a^b g(x)dx$
C.GSR.5.5	Apply the Fundamental Theorem of Calculus as an interpretation of the accumulation in the rate of change of a function as equivalent to the change in the antiderivative over the interval.	
Find the antiderivative of indefinite integrals.		
C.GSR.5.6	Apply Fundamental Theorem of Calculus to indefinite integrals to represent the family of antiderivatives.	
C.GSR.5.7	Apply integration by substitution to definite and indefinite integrals.	

PATTERNING & ALGEBRAIC REASONING – *Applications of Integrals*		
C.PAR.6: Apply the definite integral and indefinite integral to contextual situations.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
Apply Integration techniques to solve problems.		
C.PAR.6.1	Find a particular curve in a family of antiderivatives using an initial condition.	
C.PAR.6.2	Solve separable differential equations and use them to model real-world problems.	***Relevance and Application*** • Interpret slope fields as a visual representation of antiderivatives.
C.PAR.6.3	Apply definite integrals to find the area between two curves.	***Strategies and Methods*** • Include the area between a curve and the x-axis or y-axis.
C.PAR.6.4	Apply definite integrals to find the average value of a function over a closed interval.	***Relevance and Application*** • Include algebraic, numerical, and graphical contextual situations such as traffic flow, average cost, etc.

ESSENTIAL INSTRUCTIONAL GUIDANCE

MATHEMATICAL PRACTICES

The Mathematical Practices describe the reasoning behaviors students should develop as they build an understanding of mathematics – the “habits of mind” that help students become mathematical thinkers. There are eight standards, which apply to all grade levels and conceptual categories.

These mathematical practices describe how students should engage with the mathematics content for their grade level. Developing these habits of mind builds students’ capacity to become mathematical thinkers. These practices can be applied individually or together in mathematics lessons, and no particular order is required. In well-designed lessons, there are often two or more Standards for Mathematical Practice present.

Mathematical Practices	
C.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.	
Code	**Expectation**
C.MP.1	Make sense of problems and persevere in solving them.
C.MP.2	Reason abstractly and quantitatively.
C.MP.3	Construct viable arguments and critique the reasoning of others.
C.MP.4	Model with mathematics.
C.MP.5	Use appropriate tools strategically.
C.MP.6	Attend to precision.
C.MP.7	Look for and make use of structure.
C.MP.8	Look for and express regularity in repeated reasoning.

MATHEMATICAL MODELING

Teaching students to model with mathematics is engaging, builds confidence and competence, and gives students the opportunity to collaborate and make sense of the world around them, the main reason for doing mathematics. For these reasons, mathematical modeling should be incorporated at every level of a student's education. This is important not only to develop a deep understanding of mathematics itself, but more importantly to give students the tools they need to make sense of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed daily life decisions based on data and the models they create.

The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a real-life problem or task.

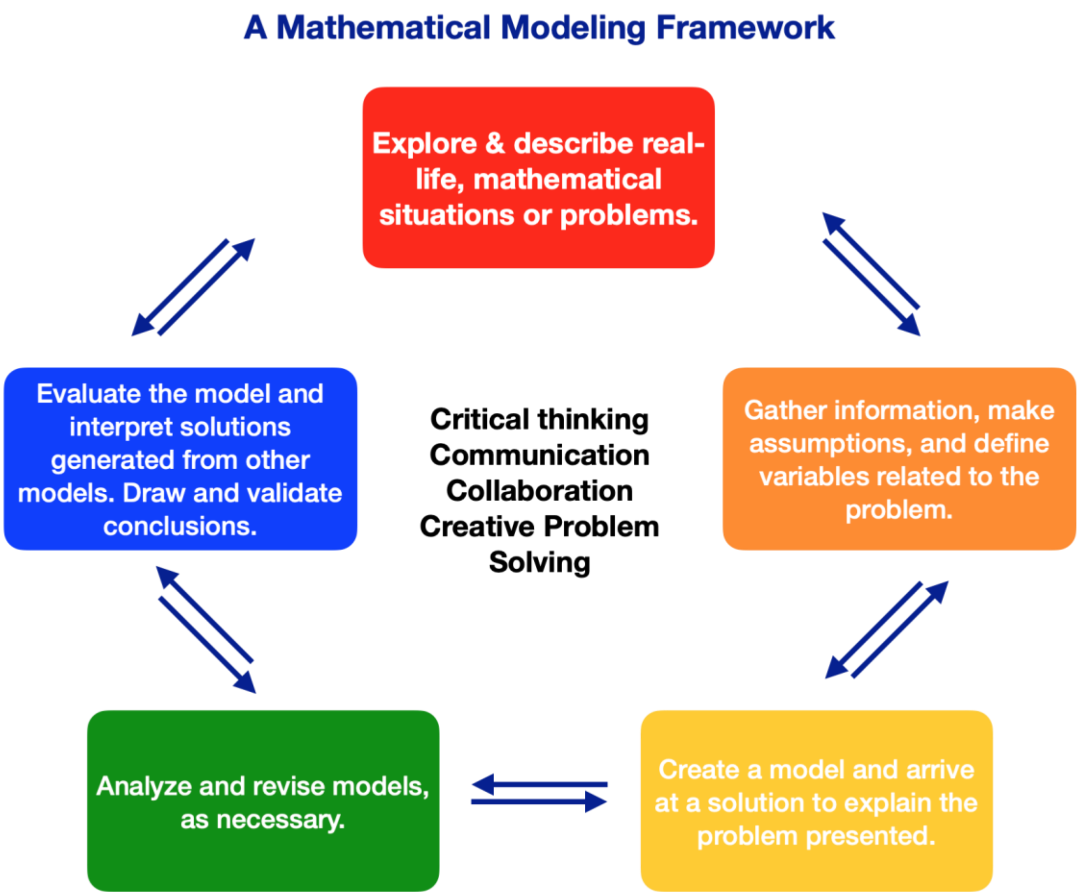

Image adapted from: Suh, Matson, Seshaiyer, 2017

FRAMEWORK FOR STATISTICAL REASONING

Statistical reasoning is important for learners to engage as citizens and professionals in a world that continues to change and evolve. Humans are naturally curious beings and statistics is a language that can be used to better answer questions about personal choices and/or make sense of naturally occurring phenomena. Statistics is a way to ask questions, explore, and make sense of the world around us.

The Framework for Statistical Reasoning should be used in all grade levels and courses to guide learners through the sense-making process, ultimately leading to the goal of statistical literacy in all grade levels and courses. Reasoning with statistics provides a context that necessitates the learning and application of a variety of mathematical concepts.

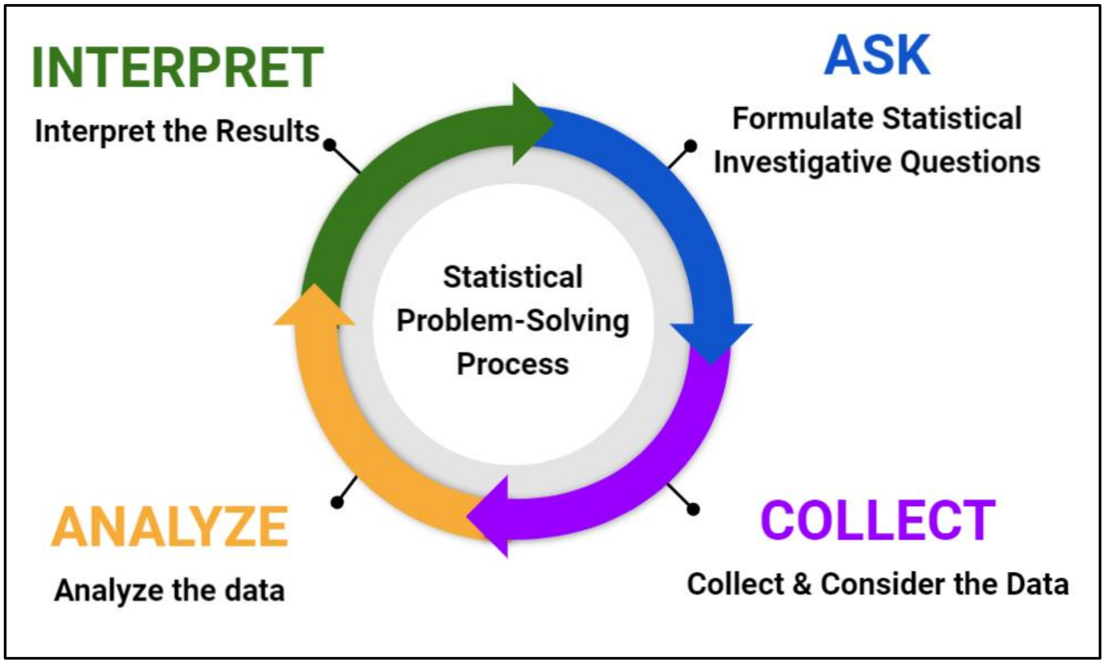

Figure 1: Georgia Framework for Statistical Reasoning

The following four-step statistical problem-solving process can be used throughout each grade level and course to help learners develop a solid foundation in statistical reasoning and literacy:

I. **Formulate Statistical Investigative Questions**
Ask questions that anticipate variability.

II. **Collect & Consider the Data**
Ensure that data collection designs acknowledge variability.

III. **Analyze the Data**
Make sense of data and communicate what the data mean using pictures (graphs) and words. Give an accounting of variability, as appropriate.

IV. **Interpret the Results**
Answer statistical investigative questions based on the collected data.