Differential Equations

MATHEMATICS KEY COMPETENCIES & COURSE STANDARDS

WITH LEARNING OBJECTIVES IN PROGRESSION ORDER

GEORGIA'S K-12 MATHEMATICS STANDARDS 2021

Governor Kemp and Superintendent Woods are committed to the best set of academic standards for Georgia's students – laying a strong foundation of the fundamentals, ensuring age- and developmentally appropriate concepts and content, providing instructional supports to set our teachers up for success, protecting and affirming local control and flexibility regarding the use of mathematical strategies and methods, and preparing students for life. These Georgia-owned and Georgia-grown standards leverage the insight, expertise, experience, and efforts of thousands of Georgians to deliver the very best educational experience for Georgia's 1.7 million students.

In August 2019, Governor Brian Kemp and State School Superintendent Richard Woods announced the review and revision of Georgia's K-12 mathematics standards. Georgians have been engaged throughout the standards review and revision process through public surveys and working groups. In addition to educator working groups, surveys, and the Academic Review Committee, Governor Kemp announced a new way for Georgians to provide input on the standards: the Citizens Review Committee, a group composed of students, parents, business and community leaders, and concerned citizens from across the state. Together, these efforts were undertaken to ensure Georgians will have buy-in and faith in the process and product.

The Citizens Review Committee provided a charge and recommendations to the working groups of educators who came together to craft the standards, ensuring the result would be usable and friendly for parents and students in addition to educators. More than 14,000 Georgians participated in the state's public survey from July through September 2019, providing additional feedback for educators to review. The process of writing the standards involved more than 200 mathematics educators -- from beginning to veteran teachers, representing rural, suburban, and metro areas of our state.

Grade-level teams of mathematics teachers engaged in deep discussions; analyzed stakeholder feedback; reviewed every single standard, concept, and skill; and provided draft recommendations. To support fellow mathematics teachers, they also developed learning progressions to show when key concepts were introduced and how they progressed across grade levels, provided examples, and defined age/developmentally appropriate expectations.

These teachers reinforced that strategies and methods for solving mathematical problems are classroom decisions -- not state decisions -- and should be made with the best interest of the individual child in mind. These recommended revisions have been shared with the Academic Review Committee, which is composed of postsecondary partners, age/development experts, and business leaders, as well as the Citizens Review Committee, for final input and feedback.

Based on the recommendation of Superintendent Woods, the State Board of Education will vote to post the draft K-12 mathematics standards for public comment. Following public comment, the standards will be recommended for adoption, followed by a year of teacher training and professional learning prior to implementation.

Differential Equations

Overview

This document contains a draft of Georgia's 2021 K-12 Mathematics Standards for the High School Differential Equations Course, which is a fourth mathematics course option in the high school course sequence.

The standards are organized into big ideas, course competencies/standards, and learning objectives/expectations. The grade level key competencies represent the standard expectation of learning for students in each grade level. The competencies/standards are each followed by more detailed learning objectives that further explain the expectations for learning in the specific grade levels.

New instructional supports are included, such as clarification of language and expectations, as well as detailed examples. These have been provided for teaching professionals and stakeholders through the Evidence of Student Learning Column that accompanies each learning objective.

Course Description:

Differential Equations is an option for students who wish to enroll in a mathematics course beyond calculus. The course introduces ordinary differential equations. Topics include the solution of first, second, and higher order differential equations, systems of differential equations, series solutions and Laplace transforms. There will be a strong focus on the presentation of mathematical ideas through both written and oral communication. The goal is to give students the skills and techniques they will need as they study advanced mathematics at the college level.

Prerequisite:

This course is designed for students who have successfully completed *AP Calculus BC*.

Georgia's K-12 Mathematics Standards - 2021
Mathematics Big Ideas and Learning Progressions, High School

Mathematics Big Ideas, HS

HIGH SCHOOL
MATHEMATICAL PRACTICES (MP)
MATHEMATICAL MODELING (MM)
NUMERICAL (QUANTITATIVE) REASONING (NR)
PATTERNING & ALGEBRAIC REASONING (PAR)
FUNCTIONAL & GRAPHICAL REASONING (FGR)
GEOMETRIC & SPATIAL REASONING (GSR)
DATA & STATISTICAL REASONING (DSR)
PROBABILISTIC REASONING (PR)
ABSTRACT REASONING (AR)

The 8 Mathematical Practices and the Mathematical Modeling Framework are essential to the implementation of the content standards presented in this course. More details related to these concepts can be found in the links below and in the first two standards presented in this course:

Mathematical Practices

Mathematical Modeling Framework

Differential Equations

The six course standards listed below are the key content competencies students will be expected to master in this course. Additional clarity and details are provided through the classroom-level learning objectives and evidence of student learning details for each course standard found on subsequent pages of this document.

COURSE STANDARDS
DE.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.
DE.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.
DE.AR.2: Solve contextual, mathematical problems involving first-order differential equations to explain real-life phenomena.
DE.AR.3: Solve contextual, mathematical problems involving second and higher order differential equations to explain real-life phenomena.
DE.AR.4: Solve contextual, mathematical problems involving systems of differential equations to explain real-life phenomena.
DE.AR.5: Solve contextual, mathematical problems using Laplace transforms to explain real-life phenomena.

Differential Equations

MATHEMATICAL MODELING		
DE.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
DE.MM.1.1	Explain contextual, mathematical problems using a mathematical model.	***Fundamentals*** • Students should be provided with opportunities to learn mathematics in the context of real-life problems. • Contextual, mathematical problems are mathematical problems presented in context where the context makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (model with mathematics).
DE.MM.1.2	Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.	***Fundamentals*** • Students should be able to use the content learned in this course to create a mathematical model to explain real-life phenomena.
DE.MM.1.3	Using abstract and quantitative reasoning, make decisions about information and data from a contextual situation.	
DE.MM.1.4	Use various mathematical representations and structures with this information to represent and solve real-life problems.	

ABSTRACT REASONING – *First Order Differential Equations*		
DE.AR.2: Solve contextual, mathematical problems involving first-order differential equations to explain real-life phenomena.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
Solve and use first order differential equations.		
DE.AR.2.1	Classify differential equations by order and linearity.	
DE.AR.2.2	Solve separable differential equations for general solutions and initial value problems.	
DE.AR.2.3	Solve first-order linear differential equations and initial value problems using integrating factors.	***Example*** • Solve $y' + 2xy = x$ using an integrating factor.
DE.AR.2.4	Use modeling or numerical methods to approximate solutions of first-order differential equations in context.	***Relevance and Application*** • Students use Euler's method or the Runge-Kutta method to find an approximate solution to a differential equation model arising from context.
DE.AR.2.5	Draw direction fields containing solutions curves for first-order differential equations by hand and using modeling.	

DE.AR.2.6	Solve first-order linear differential equations that apply to various real-world models including falling bodies, mixtures, population and the logistic equation, continuously compounded interest, and other physics applications.	

ABSTRACT REASONING *– Higher Order Differential Equations*		
DE.AR.3: Solve contextual, mathematical problems involving second and higher order differential equations to explain real-life phenomena.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
Solve and use second and higher order differential equations.		
DE.AR.3.1	Determine whether a first- or second-order differential equation has a unique solution over a given interval by applying the Existence and Uniqueness Theorem.	***Examples*** • Show that $xy' + y = x^2$ has one solution through $y(1) = 0$.
DE.AR.3.2	Solve second-order linear homogeneous and non-homogeneous differential equations by finding characteristic equations, using the method of undetermined coefficients and variation of parameters.	***Examples*** • Characteristic equations of one real root, two real roots, or two non-real roots.
DE.AR.3.3	Solve second-order differential equations that apply to various real-world models.	***Examples*** • Mass-spring systems; electric circuits; economics.
DE.AR.3.4	Use vector function notation when discussing the structure of solution sets for homogeneous systems as it pertains to the Wronskian.	
DE.AR.3.5	Determine the existence and uniqueness of solutions for second-order linear differential equations, determine a fundamental set of solutions, and verify that two solutions form a fundamental set by taking the Wronskian.	
DE.AR.3.6	Determine the structure of solution set to higher-order differential equations, apply the basic Existence and Uniqueness Theorem to higher-order differential equations, and use the generalizations of the Wronksian for higher order differential equations.	
DE.AR.3.7	Solve higher-order constant coefficient homogeneous differential equations.	
DE.AR.3.8	Solve special case non-homogeneous second order ordinary differential equations including Cauchy-Euler Equations.	

DE.AR.3.9	Find a second linearly dependent solution using reduction of order when given a solution to a non-homogeneous second-order differential equation.	
Find and use series solutions.		
DE.AR.3.10	Determine ordinary points, recurrence relations, and change the index as they relate to series solutions to ordinary differential equations.	
DE.AR.3.11	Find series solutions to first and second-order non-linear initial value problems.	

ABSTRACT REASONING *– Systems of Differential of Equations*		
DE.AR.4: Solve contextual, mathematical problems involving systems of differential equations to explain real-life phenomena.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
DE.AR.4.1	Determine whether a contextual situation results in a system of differential equations and apply the basic existence and uniqueness results for the corresponding initial value problems.	
DE.AR.4.2	Solve constant coefficient homogeneous systems using eigenvalues and eigenvectors. Solve systems with real, distinct eigenvalues, as well as those with repeated and imaginary eigenvalues.	
DE.AR.4.3	Draw phase portraits for solutions of homogeneous systems with constant coefficients.	
DE.AR.4.4	Solve non-homogeneous systems of ordinary differential equations using the method of undetermined coefficients and variation of parameters.	
DE.AR.4.5	Determine which non-linear systems are locally linear and identify the behavior of the system about each critical point.	
DE.AR.4.6	Plot locally linear systems.	
DE.AR.4.7	Use population models derived from locally linear systems.	***Examples*** • Lotka-Volterra, competition and cooperation models.

ABSTRACT REASONING *– Laplace Transforms*		
DE.AR.5: Solve contextual, mathematical problems using Laplace transforms to explain real-life phenomena.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**

DE.AR.5.1	Use the integral definition to perform Laplace transforms for functions.	***Examples*** • Polynomial, exponential, and trigonometric functions.
DE.AR.5.2	Use a Laplace table to accurately and efficiently identify Laplace transforms.	***Examples*** • The transforms for polynomials, exponentials, and trigonometric functions, and the product of these functions.
DE.AR.5.3	Perform inverse Laplace transforms using a variety of techniques.	***Examples*** • Algebraic manipulation and partial fraction decomposition.
DE.AR.5.4	Solve first- and second-order differential equations using Laplace transforms that apply to fields such as electrical and mechanical engineering.	
DE.AR.5.5	Write piecewise functions as compositions of step (Heaviside) functions.	
DE.AR.5.6	Find the general uniqueness and existence of solutions for step functions, and use Laplace transforms to find solutions to step functions.	
DE.AR.5.7	Find the Laplace transform of the Dirac delta function.	
DE.AR.5.8	Solve linear systems of differential equations using Laplace transforms.	

ESSENTIAL INSTRUCTIONAL GUIDANCE

MATHEMATICAL PRACTICES

The Mathematical Practices describe the reasoning behaviors students should develop as they build an understanding of mathematics – the "habits of mind" that help students become mathematical thinkers. There are eight standards, which apply to all grade levels and conceptual categories.

These mathematical practices describe how students should engage with the mathematics content for their grade level. Developing these habits of mind builds students' capacity to become mathematical thinkers. These practices can be applied individually or together in mathematics lessons, and no particular order is required. In well-designed lessons, there are often two or more Standards for Mathematical Practice present.

Mathematical Practices	
DE.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.	
Code	**Expectation**
DE.MP.1	Make sense of problems and persevere in solving them.
DE.MP.2	Reason abstractly and quantitatively.
DE.MP.3	Construct viable arguments and critique the reasoning of others.
DE.MP.4	Model with mathematics.
DE.MP.5	Use appropriate tools strategically.
DE.MP.6	Attend to precision.
DE.MP.7	Look for and make use of structure.
DE.MP.8	Look for and express regularity in repeated reasoning.

MATHEMATICAL MODELING

Teaching students to model with mathematics is engaging, builds confidence and competence, and gives students the opportunity to collaborate and make sense of the world around them, the main reason for doing mathematics. For these reasons, mathematical modeling should be incorporated at every level of a student's education. This is important not only to develop a deep understanding of mathematics itself, but more importantly to give students the tools they need to make sense of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed daily life decisions based on data and the models they create.

The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a real-life problem or task.

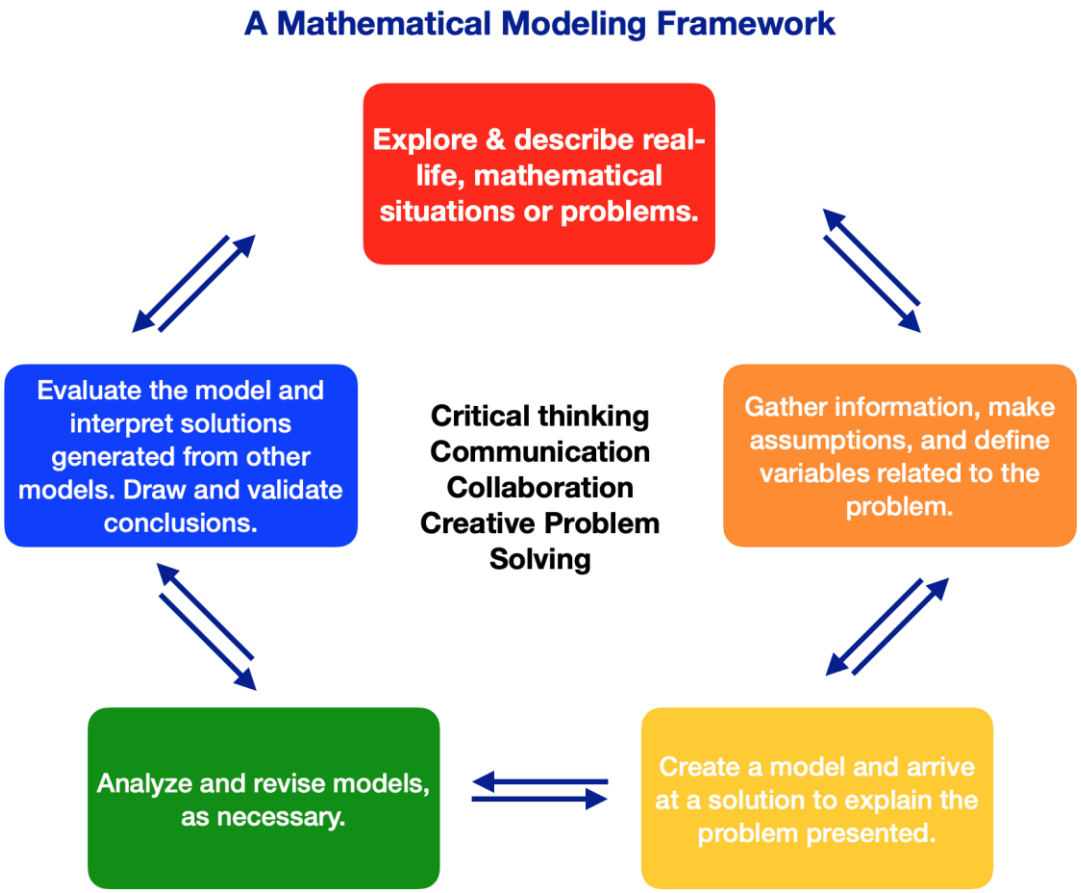

Image adapted from: Suh, Matson, Seshaiyer, 2017

FRAMEWORK FOR STATISTICAL REASONING

Statistical reasoning is important for learners to engage as citizens and professionals in a world that continues to change and evolve. Humans are naturally curious beings and statistics is a language that can be used to better answer questions about personal choices and/or make sense of naturally occurring phenomena. Statistics is a way to ask questions, explore, and make sense of the world around us.

The Framework for Statistical Reasoning should be used in all grade levels and courses to guide learners through the sense-making process, ultimately leading to the goal of statistical literacy in all grade levels and courses. Reasoning with statistics provides a context that necessitates the learning and application of a variety of mathematical concepts.

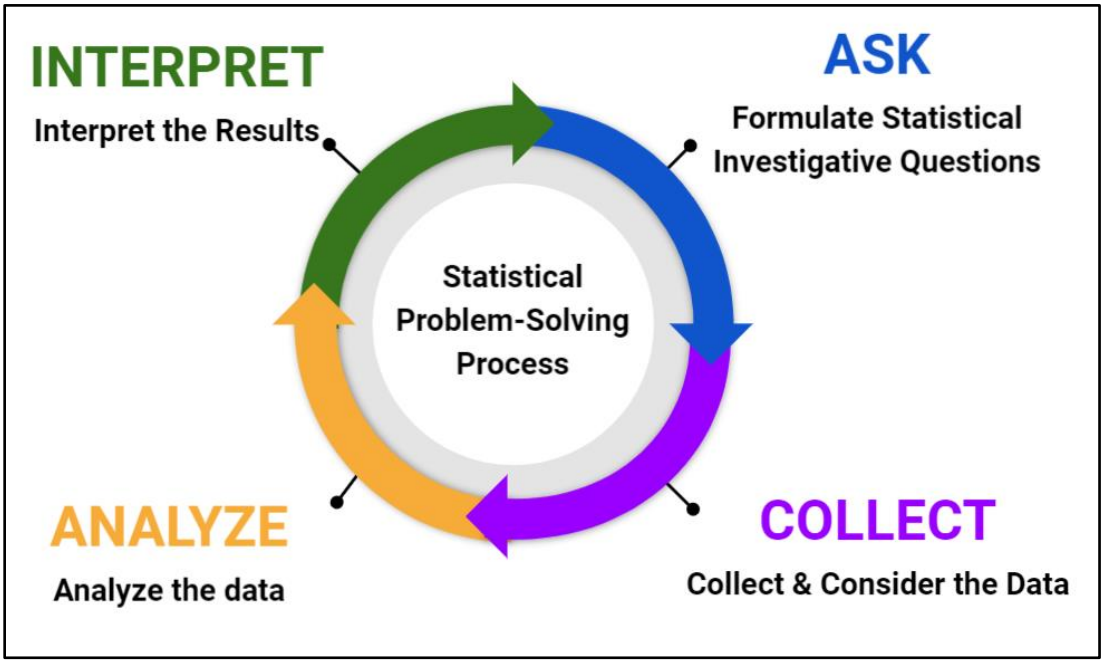

Figure 1: Georgia Framework for Statistical Reasoning

The following four-step statistical problem-solving process can be used throughout each grade level and course to help learners develop a solid foundation in statistical reasoning and literacy:

I. Formulate Statistical Investigative Questions
Ask questions that anticipate variability.

II. Collect & Consider the Data
Ensure that data collection designs acknowledge variability.

III. Analyze the Data
Make sense of data and communicate what the data mean using pictures (graphs) and words. Give an accounting of variability, as appropriate.

IV. Interpret the Results
Answer statistical investigative questions based on the collected data.