Mathematics of Industry and Government

MATHEMATICS KEY COMPETENCIES & COURSE STANDARDS

WITH LEARNING OBJECTIVES IN PROGRESSION ORDER

Governor Kemp and Superintendent Woods are committed to the best set of academic standards for Georgia's students – laying a strong foundation of the fundamentals, ensuring age- and developmentally appropriate concepts and content, providing instructional supports to set our teachers up for success, protecting and affirming local control and flexibility regarding the use of mathematical strategies and methods, and preparing students for life. These Georgia-owned and Georgia-grown standards leverage the insight, expertise, experience, and efforts of thousands of Georgians to deliver the very best educational experience for Georgia's 1.7 million students.

In August 2019, Governor Brian Kemp and State School Superintendent Richard Woods announced the review and revision of Georgia's K-12 mathematics standards. Georgians have been engaged throughout the standards review and revision process through public surveys and working groups. In addition to educator working groups, surveys, and the Academic Review Committee, Governor Kemp announced a new way for Georgians to provide input on the standards: the Citizens Review Committee, a group composed of students, parents, business and community leaders, and concerned citizens from across the state. Together, these efforts were undertaken to ensure Georgians will have buy-in and faith in the process and product.

The Citizens Review Committee provided a charge and recommendations to the working groups of educators who came together to craft the standards, ensuring the result would be usable and friendly for parents and students in addition to educators. More than 14,000 Georgians participated in the state's public survey from July through September 2019, providing additional feedback for educators to review. The process of writing the standards involved more than 200 mathematics educators -- from beginning to veteran teachers, representing rural, suburban, and metro areas of our state.

Grade-level teams of mathematics teachers engaged in deep discussions; analyzed stakeholder feedback; reviewed every single standard, concept, and skill; and provided draft recommendations. To support fellow mathematics teachers, they also developed learning progressions to show when key concepts were introduced and how they progressed across grade levels, provided examples, and defined age/developmentally appropriate expectations.

These teachers reinforced that strategies and methods for solving mathematical problems are classroom decisions -- not state decisions -- and should be made with the best interest of the individual child in mind. These recommended revisions have been shared with the Academic Review Committee, which is composed of postsecondary partners, age/development experts, and business leaders, as well as the Citizens Review Committee, for final input and feedback.

Based on the recommendation of Superintendent Woods, the State Board of Education will vote to post the draft K-12 mathematics standards for public comment. Following public comment, the standards will be recommended for adoption, followed by a year of teacher training and professional learning prior to implementation.

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Overview

This document contains a draft of Georgia's 2021 K-12 Mathematics Standards for the High School Mathematics of Industry and Government course, which is a fourth mathematics course option in the high school course sequence.

The standards are organized into big ideas, course competencies/standards, and learning objectives/expectations. The grade level key competencies represent the standard expectation of learning for students in each grade level. The competencies/standards are each followed by more detailed learning objectives that further explain the expectations for learning in the specific grade levels.

New instructional supports are included, such as clarification of language and expectations, as well as detailed examples. These have been provided for teaching professionals and stakeholders through the Evidence of Student Learning Column that accompanies each learning objective.

Course Description:

Mathematics of Industry and Government is a fourth-year mathematics course designed for students who have successfully completed Advanced Algebra / Algebra II. Modeled after operations research courses, this course allows students to explore decision making in a variety of industries such as: Airline - scheduling planes and crews, pricing tickets, taking reservations, and planning the size of the fleet; Pharmaceutical - R& D management; Logistics companies - routing and planning; Lumber and wood products - managing forests and cutting timber; Local government - deployment of emergency services, and Policy studies and regulation - environmental pollution, air traffic safety, AIDS, and criminal justice policy.

Focus is on the development of mathematical models that can be used to model, improve, predict, and optimize real-world systems. These mathematical models include both deterministic models such as mathematical programming, routing or network flows and probabilistic models such as queuing, and simulation.

phenomena.

Prerequisite:

This course is designed for students who have successfully completed *Advanced Algebra / Algebra II.*

Georgia's K-12 Mathematics Standards - 2021

Mathematics Big Ideas and Learning Progressions, High School

Mathematics Big Ideas, HS

HIGH SCHOOL
MATHEMATICAL PRACTICES (MP)
MATHEMATICAL MODELING (MM)
NUMERICAL (QUANTITATIVE) REASONING (NR)
PATTERNING & ALGEBRAIC REASONING (PAR)
FUNCTIONAL & GRAPHICAL REASONING (FGR)
GEOMETRIC & SPATIAL REASONING (GSR)
DATA & STATISTICAL REASONING (DSR)
PROBABILISTIC REASONING (PR)
ABSTRACT REASONING & DETERMINISTIC DECISION-MAKING (ARDDM)
ABSTRACT REASONING & PROBABILISTIC DECISION-MAKING (ARPDM)

The 8 Mathematical Practices and the Mathematical Modeling Framework are essential to the implementation of the content standards presented in this course. More details related to these concepts can be found in the links below and in the first two standards presented in this course:

Mathematical Practices

Mathematical Modeling Framework

Mathematics of Industry & Government

The ten course standards listed below are the key content competencies students will be expected to master in this course. Additional clarity and details are provided through the classroom-level learning objectives and evidence of student learning details for each course standard found on subsequent pages of this document.

COURSE STANDARDS
MIG.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration, and expression. Seek help and apply feedback. Set and monitor goals.
MIG.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.
MIG.ARDDM.2: Solve contextual, mathematical problems involving linear programming and use the mathematics as a model to make decisions about real life phenomena.
MIG.ARDDM.3: Solve contextual, mathematical problems involving optimal locations and use the mathematics as a model to make decisions about real life phenomena.
MIG.ARDDM.4: Solve contextual, mathematical problems involving optimal paths and use the mathematics as a model to make decisions about real life phenomena.
MIG.ARPDM.5: Solve contextual, mathematical problems with normal distributions to make appropriate decisions.
MIG.ARPDM.6: Solve contextual, mathematical problems using other distributions (e.g., binomial, geometric, and Poisson) as well as simulations to make appropriate decisions.
MIG.PR.7: Use probabilistic models to make appropriate decisions.
MIG.ARPDM.8: Use simulations to make appropriate decisions.
MIG.ARPDM.9: Using quantitative reasoning, determine fair methods to reflect the wishes of a larger population with representatives.

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MATHEMATICAL MODELING			
MIG.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.			
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**	
MIG.MM.1.1	Explain contextual, mathematical problems using a mathematical model.	***Fundamentals*** • Students should be provided with opportunities to learn mathematics in the context of real-life problems. • Contextual, mathematical problems are mathematical problems presented in context where the context makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (model with mathematics).	
MIG.MM.1.2	Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.	***Fundamentals*** • Students should be able to use the content learned in this course to create a mathematical model to explain real-life phenomena.	
MIG.MM.1.3	Using abstract and quantitative reasoning, make decisions about information and data from a contextual situation.	***Relevance and Applications*** • Students will use the multi-attribute utility theory (MAUT) to compare alternatives and handle tradeoffs based on ranking strengths and weaknesses among multiple objectives to make the most well-informed decision possible.	***Examples*** • Choosing a cell phone plan. • Choosing a college. • Buying a car, home, computer, etc.
MIG.MM.1.4	Use various mathematical representations and structures with this information to represent and solve real-life problems.	***Relevance and Application*** • Students will identify criteria and measures, collect data, find a range of each measure, rescale each measure to a common unit, conduct an interview to calculate weights, calculate total scores and interpret results to make final decision.	***Examples*** • Choosing health, auto, and/or homeowners' insurance. • Choosing a service provider – supplier, building contractor, plumber, etc. • Choosing a career.

ABSTRACT REASONING & DETERMINISTIC DECISION-MAKING *– Linear Programming*			
MIG.ARDDM.2: Solve contextual, mathematical problems involving linear programming and use the mathematics as a model to make decisions about real life phenomena.			
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**	
MIG.ARDDM.2.1	Use advanced linear programming to make decisions and interpret results in real-life contexts.	***Example*** • Optimization problems	
MIG.ARDDM.2.2	Distinguish among continuous, integer, and binary contexts	***Relevance and Application*** • Students will determine decision variables and represent them using equations or inequalities.	**Examples** • How much to make of a particular product? • How much to invest in a particular option?
MIG.ARDDM.2.3	Model and interpret results of a contextual problem with three or more variables using linear programming.	***Relevance and Application*** • Students will write an objective function that captures the goal of the problem.	***Fundamentals*** • Students will write an objective function that captures the goal of the problem (this determines when you have found the optimal solution and is the equation that you want to maximize). • Students will define the constraints – those things that limit the choices
MIG.ARDDM.2.4	Solve problems with three or more variables using technology and principles of linear programming.	***Example*** • Once students have defined the problem, the information can be entered into a spreadsheet in Excel and, using the solver parameters, solver will determine the optimal solution.	
MIG.ARDDM.2.5	Examine cause and effect of contextual changes.	**Examples** • Student will perform a sensitivity analysis to determine the effect of different changes – what if the situation changes? Will the optimal solution change and by how much? How much would the situation have to change before you would need to re-run solver?	

ABSTRACT REASONING & DETERMINISTIC DECISION-MAKING *– Optimal Locations*		
MIG.ARDDM.3: Solve contextual, mathematical problems involving optimal locations and use the mathematics as a model to make decisions about real life phenomena.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
MIG.ARDDM.3.1	Find the optimal median location in a one-dimensional context.	***Example*** • Determine the location of a hotdog stand that would minimize customers' walking distance.

MIG.ARDDM.3.2	Find the optimal median location in a rectilinear context.	***Example*** • Plots of land
MIG.ARDDM.3.3	Find the optimal location given three equally weighted, noncollinear points	***Example*** • Finding the circumcenter for a business to maximize their profits given 3 points of interest.
MIG.ARDDM.3.4	Find the optimal location in a set covering context.	***Example*** • Determine the minimum number of fire stations that satisfy certain criteria, i.e, locations of fire stations should be able to respond within a specific amount of time and can cover all residence minimizing the amount of stations to open.

ABSTRACT REASONING & DETERMINISTIC DECISION-MAKING *– Optimal Paths*		
MIG.ARDDM.4: Solve contextual, mathematical problems involving optimal paths and use the mathematics as a model to make decisions about real life phenomena.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
MIG.ARDDM4.1	Relate context to a network representation.	***Example*** • Determine the location for disaster response agencies – tornadoes, earthquakes, wildfires, etc.
MIG.ARDDM.4.2	Apply appropriate recursive algorithms.	***Examples*** • Minimum spanning tree, shortest path, critical path management
MIG.ARDDM.4.3	Examine alternate decisions in response to contextual changes.	***Example*** • Determine the expected savings from relying on wind vs oil after comparing the cost of land and turbine equipment to the price of oil production.

ABSTRACT REASONING & PROBABILISTIC DECISION-MAKING *– Normal Distributions*		
MIG.ARPDM.5: Solve contextual, mathematical problems with normal distributions to make appropriate decisions.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
MIG.ARPDM.5.1	Use properties of normal distributions to make decisions about optimization and efficiency.	***Examples*** • Battery life • Gas mileage • Pieces of candy in a bag
MIG.ARPDM.5.2	Calculate, analyze and interpret theoretical and empirical probabilities using standardized and non-standardized data.	***Terminology*** • Standardized data is that data that has been normalized to a mean of zero and a standard deviation of 1.
MIG.ARPDM.5.3	Consider contextual factors and investigate issues within the decision-making process.	***Example*** • Cost – Benefit Analysis – Does the expected profit from a product justify the cost of producing the product?

MIG.ARPDM.5.4	Apply techniques to quality control settings.	***Examples*** • Production/Assembly lines – Planes, cars, etc. • Meat packing plants • Moderating phone calls • Disposal of hazardous waste

ABSTRACT REASONING & PROBABILISTIC DECISION-MAKING *– Binomial, Geometric, and Poisson Distributions*		
MIG.ARPDM.6: Solve contextual, mathematical problems using other distributions (e.g., binomial, geometric, and Poisson) as well as simulations to make appropriate decisions.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
MIG.ARPDM.6.1	Calculate theoretical and empirical probabilities using standardized and non-standardized data.	***Examples*** • Determine the probability of making a sale • Reliability of 911 phone calls (received and answered vs not getting through)
MIG.ARPDM.6.2	Analyze and interpret the probabilities in terms of context.	***Examples*** • Determining when to add an extra worker • Determining the cost of overtime • Scheduling patients at a health care facility
MIG.ARPDM.6.3	Consider contextual factors and investigate issues within the decision-making process.	***Example*** • Flipping a house

PROBABILISTIC REASONING *– Probabilistic Models*		
MIG.PR.7: Use probabilistic models to make appropriate decisions.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
MIG.PR.7.1	Use program evaluation review technique (PERT) to investigate completion times of a project.	***Example*** • Construction of cell phone tower
MIG.PR.7.2	Develop and apply transition matrices to make predictions using Markov Chains.	***Examples*** • Cruise control systems in motor vehicles • Currency exchange rates • Animal population dynamics
MIG.PR.7.3	Apply queuing theory	***Examples*** • Customer complaints, airport security screening, DDS
MIG.PR.7.4	Consider contextual factors and investigate issues within the decision-making process.	***Example*** • Installation of google fiber networks

ABSTRACT REASONING & PROBABILISTIC DECISION-MAKING *– Simulations*		
MIG.ARPDM.8: Use simulations to make appropriate decisions.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
MIG.ARPDM.8.1	Use technology to simulate a real-world situation.	***Examples*** • Newspaper to press, Super Bowl conference dominance, customer service
MIG.ARPDM.8.2	Analyze, evaluate, and interpret results of simulations.	***Examples*** • Newspaper to press, Super Bowl conference dominance, customer service
MIG.ARPDM.8.3	Examine alternate decisions in response to contextual changes of simulations.	***Examples*** • Newspaper to press, Super Bowl conference dominance, customer service

ABSTRACT REASONING & PROBABILISTIC DECISION-MAKING *– Fair Representation*		
MIG.ARPDM.9: Using quantitative reasoning, determine fair methods to reflect the wishes of a larger population with representatives.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
MIG.ARPDM.9.1	Develop and analyze fair methods for voting.	***Examples*** • *Borda count, plurality and pairwise comparisons*
MIG.ARPDM.9.2	Develop and analyze fair methods for apportioning representatives.	***Examples*** • **Hamilton's** method, Jefferson's method
MIG.ARPDM.9.3	Develop fair methods for setting voting district boundaries.	***Example*** • Gerrymandering

ESSENTIAL INSTRUCTIONAL GUIDANCE

MATHEMATICAL PRACTICES

The Mathematical Practices describe the reasoning behaviors students should develop as they build an understanding of mathematics – the "habits of mind" that help students become mathematical thinkers. There are eight standards, which apply to all grade levels and conceptual categories.

These mathematical practices describe how students should engage with the mathematics content for their grade level. Developing these habits of mind builds students' capacity to become mathematical thinkers. These practices can be applied individually or together in mathematics lessons, and no particular order is required. In well-designed lessons, there are often two or more Standards for Mathematical Practice present.

Mathematical Practices	
MIG.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.	
Code	**Expectation**
MIG.MP.1	Make sense of problems and persevere in solving them.
MIG.MP.2	Reason abstractly and quantitatively.
MIG.MP.3	Construct viable arguments and critique the reasoning of others.
MIG.MP.4	Model with mathematics.
MIG.MP.5	Use appropriate tools strategically.
MIG.MP.6	Attend to precision.
MIG.MP.7	Look for and make use of structure.
MIG.MP.8	Look for and express regularity in repeated reasoning.

MATHEMATICAL MODELING

Teaching students to model with mathematics is engaging, builds confidence and competence, and gives students the opportunity to collaborate and make sense of the world around them, the main reason for doing mathematics. For these reasons, mathematical modeling should be incorporated at every level of a student's education. This is important not only to develop a deep understanding of mathematics itself, but more importantly to give students the tools they need to make sense of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed daily life decisions based on data and the models they create.

The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a real-life problem or task.

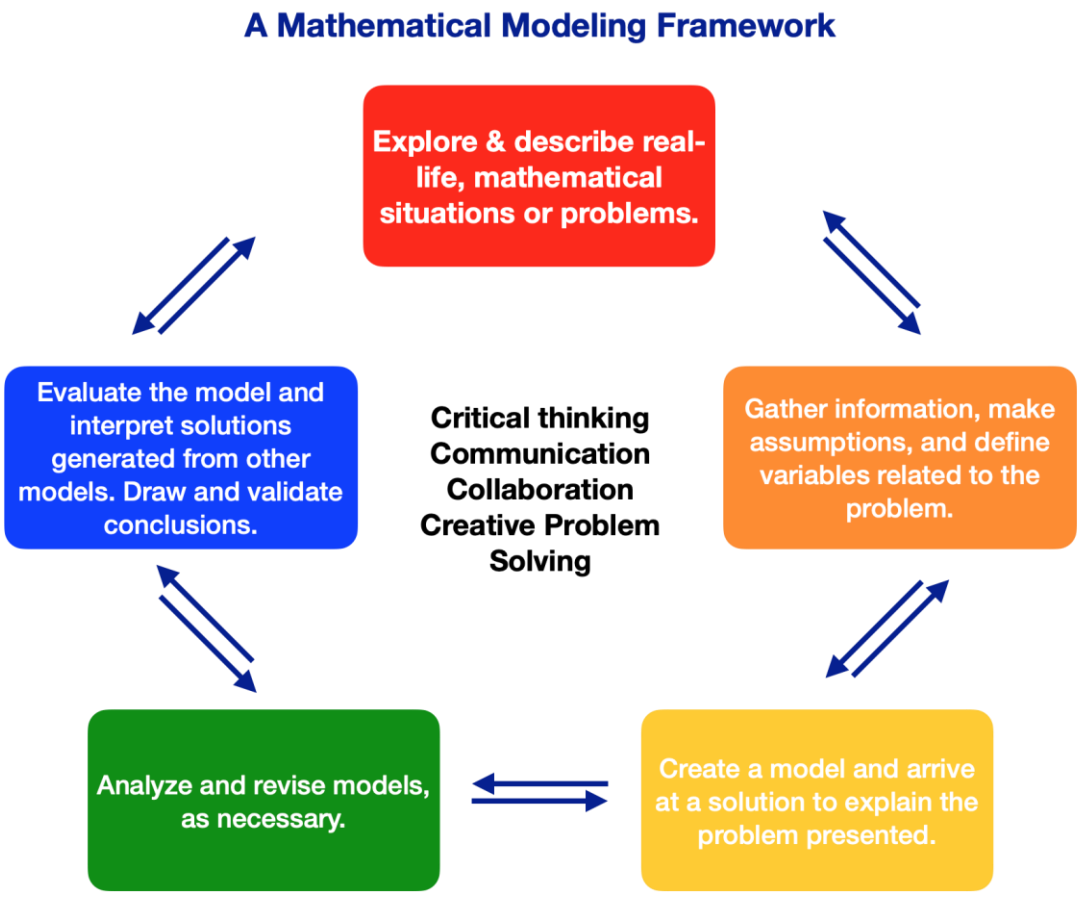

Image adapted from: Suh, Matson, Seshaiyer, 2017

FRAMEWORK FOR STATISTICAL REASONING

Statistical reasoning is important for learners to engage as citizens and professionals in a world that continues to change and evolve. Humans are naturally curious beings and statistics is a language that can be used to better answer questions about personal choices and/or make sense of naturally occurring phenomena. Statistics is a way to ask questions, explore, and make sense of the world around us.

The Framework for Statistical Reasoning should be used in all grade levels and courses to guide learners through the sense-making process, ultimately leading to the goal of statistical literacy in all grade levels and courses. Reasoning with statistics provides a context that necessitates the learning and application of a variety of mathematical concepts.

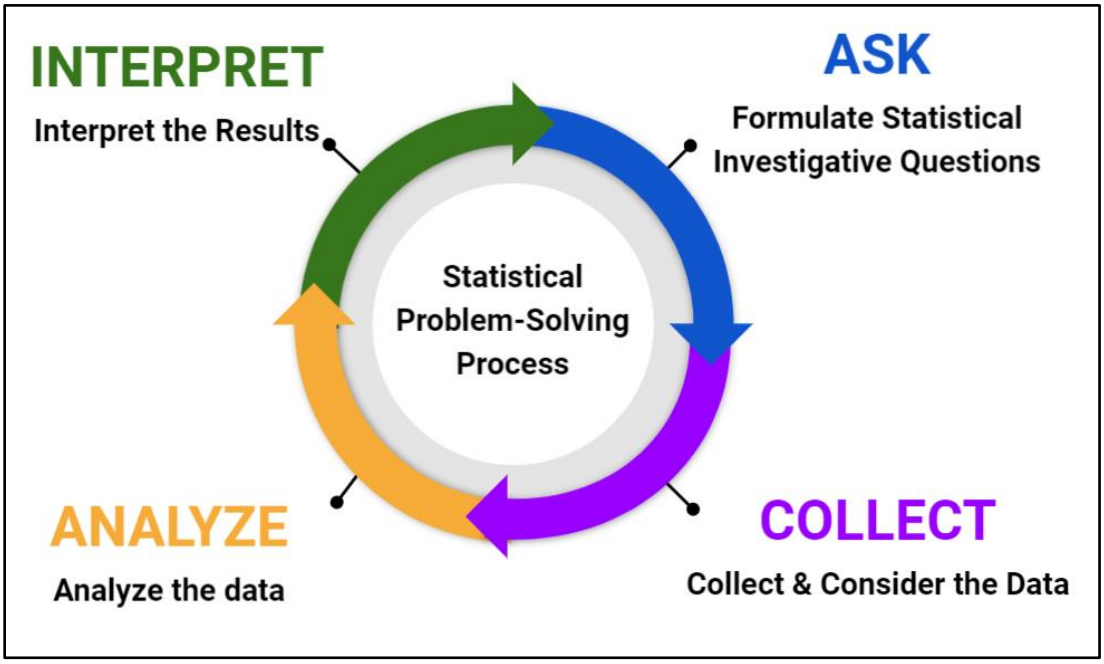

Figure 1: Georgia Framework for Statistical Reasoning

The following four-step statistical problem-solving process can be used throughout each grade level and course to help learners develop a solid foundation in statistical reasoning and literacy:

I. Formulate Statistical Investigative Questions
Ask questions that anticipate variability.

II. Collect & Consider the Data
Ensure that data collection designs acknowledge variability.

III. Analyze the Data
Make sense of data and communicate what the data mean using pictures (graphs) and words. Give an accounting of variability, as appropriate.

IV. Interpret the Results
Answer statistical investigative questions based on the collected data.