Advanced Financial Algebra

MATHEMATICS KEY COMPETENCIES & COURSE STANDARDS

WITH LEARNING OBJECTIVES IN PROGRESSION ORDER

GEORGIA'S K-12 MATHEMATICS STANDARDS 2021

Governor Kemp and Superintendent Woods are committed to the best set of academic standards for Georgia's students – laying a strong foundation of the fundamentals, ensuring age- and developmentally appropriate concepts and content, providing instructional supports to set our teachers up for success, protecting and affirming local control and flexibility regarding the use of mathematical strategies and methods, and preparing students for life. These Georgia-owned and Georgia-grown standards leverage the insight, expertise, experience, and efforts of thousands of Georgians to deliver the very best educational experience for Georgia's 1.7 million students.

In August 2019, Governor Brian Kemp and State School Superintendent Richard Woods announced the review and revision of Georgia's K-12 mathematics standards. Georgians have been engaged throughout the standards review and revision process through public surveys and working groups. In addition to educator working groups, surveys, and the Academic Review Committee, Governor Kemp announced a new way for Georgians to provide input on the standards: the Citizens Review Committee, a group composed of students, parents, business and community leaders, and concerned citizens from across the state. Together, these efforts were undertaken to ensure Georgians will have buy-in and faith in the process and product.

The Citizens Review Committee provided a charge and recommendations to the working groups of educators who came together to craft the standards, ensuring the result would be usable and friendly for parents and students in addition to educators. More than 14,000 Georgians participated in the state's public survey from July through September 2019, providing additional feedback for educators to review. The process of writing the standards involved more than 200 mathematics educators -- from beginning to veteran teachers, representing rural, suburban, and metro areas of our state.

Grade-level teams of mathematics teachers engaged in deep discussions; analyzed stakeholder feedback; reviewed every single standard, concept, and skill; and provided draft recommendations. To support fellow mathematics teachers, they also developed learning progressions to show when key concepts were introduced and how they progressed across grade levels, provided examples, and defined age/developmentally appropriate expectations.

These teachers reinforced that strategies and methods for solving mathematical problems are classroom decisions -- not state decisions -- and should be made with the best interest of the individual child in mind. These recommended revisions have been shared with the Academic Review Committee, which is composed of postsecondary partners, age/development experts, and business leaders, as well as the Citizens Review Committee, for final input and feedback.

Based on the recommendation of Superintendent Woods, the State Board of Education will vote to post the draft K-12 mathematics standards for public comment. Following public comment, the standards will be recommended for adoption, followed by a year of teacher training and professional learning prior to implementation.

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Overview

This document contains a draft of Georgia's 2021 K-12 Mathematics Standards for the High School Advanced Financial Algebra Course, which is a fourth mathematics course option in the high school course sequence.

The standards are organized into big ideas, course competencies/standards, and learning objectives/expectations. The grade level key competencies represent the standard expectation of learning for students in each grade level. The competencies/standards are each followed by more detailed learning objectives that further explain the expectations for learning in the specific grade levels.

New instructional supports are included, such as clarification of language and expectations, as well as detailed examples. These have been provided for teaching professionals and stakeholders through the Evidence of Student Learning Column that accompanies each learning objective.

Course Description:

Advanced Financial Algebra is a fourth-year mathematics course designed for students who have successfully completed Algebra II. The course extends and deepens student understanding of algebra, statistics, and research design while introducing students to relevant financial and business applications. Students will create, apply, and interpret a wide variety of algebraic function-models to aid in real-world decision making. Statistical research and analysis will be utilized to determine the efficacy of model applications and further assist in exploring scenarios with financial implications. Financial contexts for these mathematical concepts will include business operations and optimization, tax considerations, insurance and risk management, banking services, budget creation, loan and credit analysis, investment strategies and retirement plans, stock market performance, real estate fundamentals, and automobile ownership.

Instruction and assessment should include the appropriate use of manipulatives and technology. Topics should be represented in multiple ways, such as concrete/pictorial, verbal/written, numeric/data-based, graphical, and symbolic. Concepts should be introduced and used, where appropriate, in the context of realistic phenomena.

Prerequisites:

This course is designed for students who have successfully completed *Advanced Algebra* / *Algebra II*.

Georgia's K-12 Mathematics Standards - 2021
Mathematics Big Ideas and Learning Progressions, High School

Mathematics Big Ideas, HS

HIGH SCHOOL
MATHEMATICAL PRACTICES (MP)
MATHEMATICAL MODELING (MM)
NUMERICAL (QUANTITATIVE) REASONING (NR)
PATTERNING & ALGEBRAIC REASONING (PAR)
FUNCTIONAL & GRAPHICAL REASONING (FGR)
GEOMETRIC & SPATIAL REASONING (GSR)
DATA & STATISTICAL REASONING (DSR)
PROBABILISTIC REASONING (PR)

The 8 Mathematical Practices and the Mathematical Modeling Framework are essential to the implementation of the content standards presented in this course. More details related to these concepts can be found in the links below and in the first two standards presented in this course:

Mathematical Practices

Mathematical Modeling Framework

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The nine course standards listed below are the key content competencies students will be expected to master in this course. Additional clarity and details are provided through the classroom-level learning objectives and evidence of student learning details for each course standard found on subsequent pages of this document.

COURSE STANDARDS
AFA.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration, and expression. Seek help and apply feedback. Set and monitor goals.
AFA.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.
AFA.NR.2: Utilize fractions, decimals, percents, and ratios to write and solve a variety of financial problems.
AFA.FGR.3: Explore and apply functions to model and explain real-life phenomena and to solve complex problems in business and financial contexts.
AFA.PAR.4: Explore, evaluate, and rearrange formulas applicable to business and financial contexts.
AFA.PAR.5: Write and solve systems of equations and/or inequalities in context of financial applications.
AFA.GSR.6: Apply properties of polygons, circles, and trigonometry to model and explore real-world applications.
AFA.DSR.7: Collect, analyze, interpret, summarize, and construct displays of data to make predictions within real-world applications.
AFA.DSR.8: Conduct investigative research to solve real-life problems and answer statistical questions involved in business and financial decision-making.

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MATHEMATICAL MODELING		
AFA.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
AFA.MM.1.1	Explain contextual, mathematical problems using a mathematical model.	***Fundamentals*** • Students should be provided with opportunities to learn mathematics in the context of real-life problems. • Contextual, mathematical problems are mathematical problems presented in context where the context makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (model with mathematics).
AFA.MM.1.2	Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.	***Fundamentals*** • Students should be able to use the content learned in this course to create a mathematical model to explain real-life phenomena.
AFA.MM.1.3	Using abstract and quantitative reasoning, make decisions about information and data from a contextual situation.	
AFA.MM.1.4	Use various mathematical representations and structures with this information to represent and solve real-life problems.	

NUMERICAL (QUANTITATIVE) REASONING *– Fractions, Decimals, Percents, and Ratios*		
AFA.NR.2: Utilize fractions, decimals, percents, and ratios to write and solve a variety of financial problems.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
AFA.NR.2.1	Use fractions, decimals, percents, and ratios to solve problems related to budgets, income tax rates, payroll deductions, pie charts, percent yield, sales tax, percent populations, rent increase, cost savings, debt-to-income ratios, stock splits, floor plans and scale models, trigonometric calculations, banking services, and other business and financial applications.	
AFA.NR.2.2	Convert numerical quantities of one form (fractions, decimals, percents) to another within financial applications.	***Examples*** • Interest rates, depreciation rates and factors, mortgage points, etc.
AFA.NR.2.3	Calculate and interpret percent of increase and decrease.	***Examples*** • Stock yields, capital gains, budget analysis, etc.
AFA.NR.2.4	Construct, solve, and interpret algebraic ratios and proportions.	***Examples*** • Scale drawings, stock splits, etc.

FUNCTIONAL & GRAPHICAL REASONING *– Linear, Exponential, Quadratic, Cubic, Rational, Square Root, Greatest Integer, and Piecewise Functions*			
AFA.FGR.3: Explore and apply functions to model and explain real-life phenomena and to solve complex problems in business and financial contexts.			
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**	
AFA.FGR.3.1	Examine and identify the key characteristics of functions that model financial situations given the parameters of the context.	***Fundamentals*** • Students should be able to examine models of financial situations and interpret characteristics such as intercepts, intervals of increase and decrease, rates of change, positive/negative intervals, end behaviors, symmetry, relative extrema, asymptotes, jump discontinuities, and cusps.	
AFA.FGR.3.2	Solve financial problems given the parameters of the applicable context using a variety of functions.	***Fundamentals*** • Students should be able to use linear, exponential, quadratic, cubic, rational, square roots, greatest integer, and piecewise functions to solve financial problems.	
AFA.FGR.3.3	Describe the meaning of functions and how to determine if a relation is a function or not.		
AFA.FGR.3.4	Utilize function notation to represent a functional relation and to evaluate functions.		
AFA.FGR.3.5	Create, apply, and interpret linear functions to model real-world financial problems.		
AFA.FGR.3.6	Create, apply, and interpret exponential functions of the form $y = ab^x$ and classify them as exponential decay (when $0 < b < 1$) or as exponential growth (when $b > 1$).	***Examples*** • Depreciation, investment returns, etc.	
AFA.FGR.3.7	Create, apply, and interpret quadratic functions to model real-world financial applications.	***Examples*** • Business revenue and profit optimization applications.	
AFA.FGR.3.8	Create, apply, and interpret the greatest integer function in real-world financial applications.	***Fundamentals*** • This is the first-time students will be formally exposed to greatest integer functions. • Students should develop an understanding of greatest integer functions to apply them in financial situations.	***Examples*** • Cell phone data usage, widgets manufactured, etc.
AFA.FGR.3.9	Create, apply, and interpret piecewise functions in real-world financial applications.	***Fundamentals*** • This is the first-time students will be formally exposed to piecewise functions. • Students should develop an understanding of piecewise functions to apply them in financial situations.	***Examples*** • Cell phone charges, advertising costs, admission fees, etc.

AFA.FGR.3.10	Recognize real-world situations where square root, cubic, or rational functions apply.	***Examples*** • The role of skid mark structure in automobile accident-scene reconstruction, utility average cost, etc.
AFA.FGR.3.11	Create and use inequalities to define domains when creating algebraic expressions and functions.	

PATTERNING & ALGEBRAIC REASONING *– Formulas*		
AFA.PAR.4: Explore, evaluate, and rearrange formulas applicable to business and financial contexts.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
AFA.PAR.4.1	Use and rearrange formulas applicable to real-world contexts.	***Examples*** • Accident reconstruction scenes, interest calculation, area and volume, etc.
AFA.PAR.4.2	Investigate the impact of changing the value of the different variables in financial formulas to compare the resulting financial outcomes.	***Examples*** • Compare the monthly payment and total finance charge for a home loan using different down payment, principal, and interest rate values. • Compare the account balance of a periodic investment account depending upon the initial principle, deposit amount, and interest rate.
AFA.PAR.4.3	Write algebraic formulas for use in spreadsheets and utilize technology to perform both iterate and formulaic calculations.	***Examples*** • Loan amortization tables and payoff schedules, compound interest returns, etc.
AFA.PAR.4.4	Use the simple interest formula, I = Prt, and inverse operations to solve for specified variables in banking services applications and other interest problems.	
AFA.PAR.4.5	Demonstrate by iteration (both with technology and without) that the compounding process pays “interest on your interest.”	
AFA.PAR.4.6	Derive the compound interest formula, $A = P(1 + \frac{r}{t})^{nt}$, by using patterns and inductive reasoning, then compute compound interest with and without the formula.	
AFA.PAR.4.7	Explore the concept of limits of rational functions in discovering the compound continuous formula. Use technology to investigate and verify what happens as the number of compounds approaches infinity.	
AFA.PAR.4.8	Apply the natural base e in the continuous compounding formula, $A = Pe^{rt}$.	
AFA.PAR.4.9	Use the monthly payment formula to calculate payment amounts in a variety of circumstances.	***Examples*** • Applications related to financing of homes, automobiles, household appliances and furniture, student loan payments, and consumer electronics.

AFA.PAR.4.10	Utilize the monthly payment formula to assist in calculating the total interest paid (finance charge) when using credit. Compare the total of monthly payments to the original (cash) price.	
AFA.PAR.4.11	Interpret and use sigma notation.	***Examples*** • Summation of total expenses, average daily balance of credit cards, etc.
AFA.PAR.4.12	Explore and identify how the elements of the present value of a single deposit formula and the periodic deposit investment formula relate to the compound interest formula.	
AFA.PAR.4.13	Utilize the present and future value of a periodic investment formulas to make calculations regarding long-term investments and retirement planning.	

PATTERNING & ALGEBRAIC REASONING *– Systems of Equations and Inequalities*		
AFA.PAR.5: Write and solve systems of equations and/or inequalities in context of financial applications.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
AFA.PAR.5.1	Write, graph, solve, and interpret systems of linear equations given an applicable financial situation.	***Examples*** • Comparison of two cell phone plans, breakeven point of linear expense and revenue equations, etc.
AFA.PAR.5.2	Write, graph, solve, and interpret systems of equations containing one linear and one quadratic equation, given an applicable financial situation.	***Examples*** • Profit optimization, breakeven points of linear expense and quadratic revenue equations, etc.
AFA.PAR.5.3	Write, graph, and interpret systems of equations containing one linear and one exponential equation, given an applicable financial situation.	***Examples*** • Simple versus compound interest comparisons, automobile depreciation versus expenses, etc. ***Terminology*** • Linear-exponential systems have one linear equation and one exponential equation.
AFA.PAR.5.4	Write, graph, and interpret systems of a linear and a piecewise function.	***Examples*** • Comparing flat tax versus. progressive tax applications, cell phone data plans, etc.
AFA.PAR.5.5	Solve linear systems of equations and inequalities to identify points of intersection and define domains in the context of the problem situation.	

GEOMETRIC & SPATIAL REASONING *– Polygons, Circles, and Trigonometry*		
AFA.GSR.6: Apply properties of polygons, circles, and trigonometry to model and explore real-world applications.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
AFA.GSR.6.1	Apply concepts of area, volume, and scale factors to a variety of real-world financial applications.	***Examples*** • Scale drawings, carpeting, landscaping, travel, etc.
AFA.GSR.6.2	Use factors of dilations to draw to scale in contextual situations.	***Examples*** • Floor plans, survey maps, topography, etc.
AFA.GSR.6.3	Use sectors and central angles of a circle to depict proportional categories on a pie chart when given categorical information.	***Examples*** • In display of budgets and/or expenses.
AFA.GSR.6.4	Solve problems using the Pythagorean Theorem and trigonometric functions and their inverses in context.	***Examples*** • Roof pitch, wheelchair ramps, landscaping, etc.

DATA & STATISTICAL REASONING *– Data Displays*		
AFA.DSR.7: Collect, analyze, interpret, summarize, and construct displays of data to make predictions within real-world applications.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
AFA.DSR.7.1	Interpret measures of central tendency (mean, median, mode) and spread (range, interquartile range, variance, standard deviation) to analyze contextualized data sets.	***Examples*** • In the context of business expenses and revenue, personal budgets, and self-designed research projects, etc.
AFA.DSR.7.2	Construct and interpret common data displays (bar graphs, line graphs, stock bar charts, candlestick charts, box and whisker plots, stem and leaf plots, and circle graphs) to recognize and interpret trends.	***Examples*** • Stock market data, retirement planning, budgeting expenses, etc.
AFA.DSR.7.3	Construct and interpret scatterplots to recognize and interpret trends.	
AFA.DSR.7.4	Use technology to find, interpret, and graph linear, quadratic, and exponential regression equations to make predictions about the corresponding context.	
AFA.DSR.7.5	Use technology to determine the correlation coefficient of linear, quadratic, and exponential regression curves.	
AFA.DSR.7.6	Distinguish between causation and correlation for bivariate data.	
AFA.DSR.7.7	Create and analyze discrete probability distributions.	
AFA.DSR.7.8	Apply the Arithmetic Average Formula to calculate and interpret a d-day simple moving average given a set of n data points, p_1, p_2, p_3, ..., p_{n-1}, p_n.	

DATA & STATISTICAL REASONING *– Investigative Research*		
AFA.DSR.8: Conduct investigative research to solve real-life problems and answer statistical questions involved in business and financial decision-making.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
AFA.DSR.8.1	Identify a contextual, real-life problem that can be answered using investigative research.	
AFA.DSR.8.2	Develop statistical questions that can help solve a real-life problem involved in business and financial decision-making.	
AFA.DSR.8.3	Create a statistical study using sound methodology to answer statistical questions and to solve the real-life problem.	
AFA.DSR.8.4	Explain how the sample size impacts the precision with which estimates of the population parameters can be made.	***Example*** • Increasing the sample size increases the precision of the estimate.
AFA.DSR.8.5	Recognize that random selection from a population plays a different role than random assignment in an experiment.	
AFA.DSR.8.6	Incorporate random designs in data collection.	
AFA.DSR.8.7	Describe ways in which “big data” can be used to make decisions in various business enterprises and in the context of business and financial decision-making.	
AFA.DSR.8.8	Use distributions to identify the key features of the data collected.	
AFA.DSR.8.9	Interpret results and make connections to the original research question.	

ESSENTIAL INSTRUCTIONAL GUIDANCE

MATHEMATICAL PRACTICES

The Mathematical Practices describe the reasoning behaviors students should develop as they build an understanding of mathematics – the "habits of mind" that help students become mathematical thinkers. There are eight standards, which apply to all grade levels and conceptual categories.

These mathematical practices describe how students should engage with the mathematics content for their grade level. Developing these habits of mind builds students' capacity to become mathematical thinkers. These practices can be applied individually or together in mathematics lessons, and no particular order is required. In well-designed lessons, there are often two or more Standards for Mathematical Practice present.

MATHEMATICAL PRACTICES	
AFA.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.	
Code	**Expectation**
AFA.MP.1	Make sense of problems and persevere in solving them.
AFA.MP.2	Reason abstractly and quantitatively.
AFA.MP.3	Construct viable arguments and critique the reasoning of others.
AFA.MP.4	Model with mathematics.
AFA.MP.5	Use appropriate tools strategically.
AFA.MP.6	Attend to precision.
AFA.MP.7	Look for and make use of structure.
AFA.MP.8	Look for and express regularity in repeated reasoning.

MATHEMATICAL MODELING

Teaching students to model with mathematics is engaging, builds confidence and competence, and gives students the opportunity to collaborate and make sense of the world around them, the main reason for doing mathematics. For these reasons, mathematical modeling should be incorporated at every level of a student's education. This is important not only to develop a deep understanding of mathematics itself, but more importantly to give students the tools they need to make sense of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed daily life decisions based on data and the models they create.

The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a real-life problem or task.

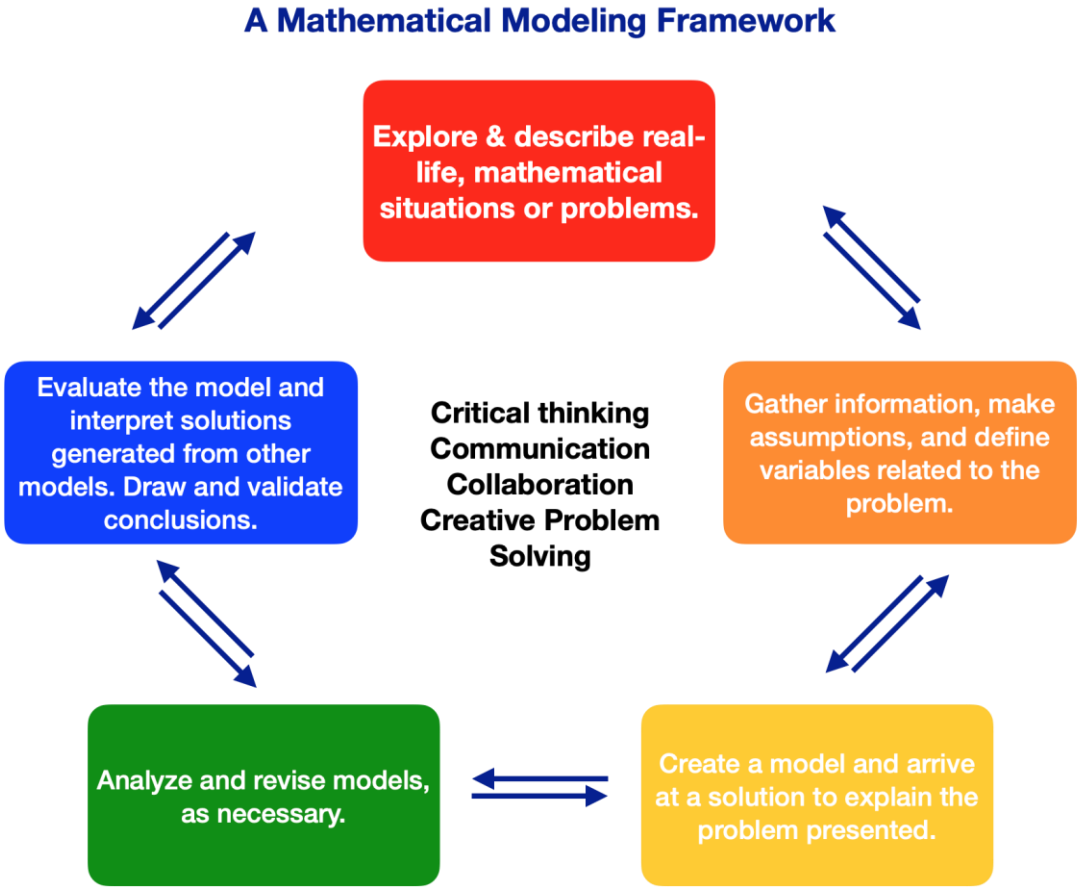

Image adapted from: Suh, Matson, Seshaiyer, 2017

FRAMEWORK FOR STATISTICAL REASONING

Statistical reasoning is important for learners to engage as citizens and professionals in a world that continues to change and evolve. Humans are naturally curious beings and statistics is a language that can be used to better answer questions about personal choices and/or make sense of naturally occurring phenomena. Statistics is a way to ask questions, explore, and make sense of the world around us.

The Framework for Statistical Reasoning should be used in all grade levels and courses to guide learners through the sense-making process, ultimately leading to the goal of statistical literacy in all grade levels and courses. Reasoning with statistics provides a context that necessitates the learning and application of a variety of mathematical concepts.

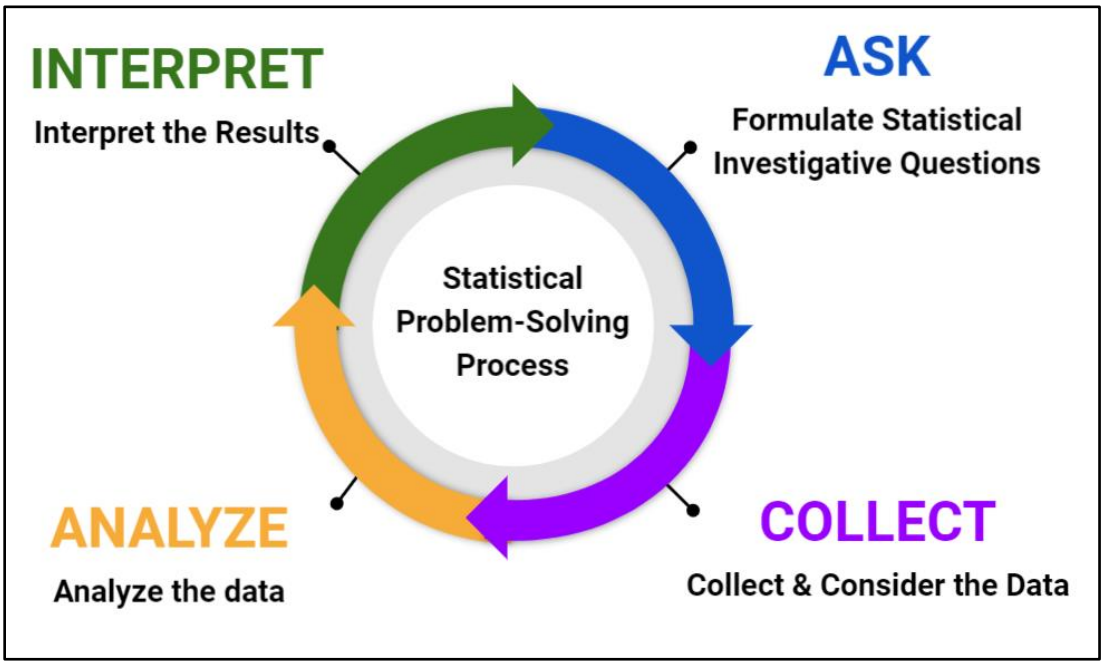

Figure 1: Georgia Framework for Statistical Reasoning

The following four-step statistical problem-solving process can be used throughout each grade level and course to help learners develop a solid foundation in statistical reasoning and literacy:

I. Formulate Statistical Investigative Questions
Ask questions that anticipate variability.

II. Collect & Consider the Data
Ensure that data collection designs acknowledge variability.

III. Analyze the Data
Make sense of data and communicate what the data mean using pictures (graphs) and words. Give an accounting of variability, as appropriate.

IV. Interpret the Results
Answer statistical investigative questions based on the collected data.