

# ***College Readiness Mathematics (Mathematics Capstone Course)***

## **MATHEMATICS KEY COMPETENCIES & COURSE STANDARDS *WITH* *LEARNING OBJECTIVES IN PROGRESSION ORDER***

# GEORGIA'S K-12 MATHEMATICS STANDARDS 2021

Governor Kemp and Superintendent Woods are committed to the best set of academic standards for Georgia's students – laying a strong foundation of the fundamentals, ensuring age- and developmentally appropriate concepts and content, providing instructional supports to set our teachers up for success, protecting and affirming local control and flexibility regarding the use of mathematical strategies and methods, and preparing students for life. These Georgia-owned and Georgia-grown standards leverage the insight, expertise, experience, and efforts of thousands of Georgians to deliver the very best educational experience for Georgia's 1.7 million students.

In August 2019, Governor Brian Kemp and State School Superintendent Richard Woods announced the review and revision of Georgia's K-12 mathematics standards. Georgians have been engaged throughout the standards review and revision process through public surveys and working groups. In addition to educator working groups, surveys, and the Academic Review Committee, Governor Kemp announced a new way for Georgians to provide input on the standards: the Citizens Review Committee, a group composed of students, parents, business and community leaders, and concerned citizens from across the state. Together, these efforts were undertaken to ensure Georgians will have buy-in and faith in the process and product.

The Citizens Review Committee provided a charge and recommendations to the working groups of educators who came together to craft the standards, ensuring the result would be usable and friendly for parents and students in addition to educators. More than 14,000 Georgians participated in the state's public survey from July through September 2019, providing additional feedback for educators to review. The process of writing the standards involved more than 200 mathematics educators -- from beginning to veteran teachers, representing rural, suburban, and metro areas of our state.

Grade-level teams of mathematics teachers engaged in deep discussions; analyzed stakeholder feedback; reviewed every single standard, concept, and skill; and provided draft recommendations. To support fellow mathematics teachers, they also developed learning progressions to show when key concepts were introduced and how they progressed across grade levels, provided examples, and defined age/developmentally appropriate expectations.

These teachers reinforced that strategies and methods for solving mathematical problems are classroom decisions -- not state decisions -- and should be made with the best interest of the individual child in mind. These recommended revisions have been shared with the Academic Review Committee, which is composed of postsecondary partners, age/development experts, and business leaders, as well as the Citizens Review Committee, for final input and feedback.

Based on the recommendation of Superintendent Woods, the State Board of Education will vote to post the draft K-12 mathematics standards for public comment. Following public comment, the standards will be recommended for adoption, followed by a year of teacher training and professional learning prior to implementation.

# College Readiness Mathematics (Mathematics Capstone Course)

## Overview

This document contains a draft of Georgia's 2021 K-12 Mathematics Standards for the High School College Readiness Mathematics Capstone Course (CRM), which is a fourth mathematics course option in the high school course sequence.

The standards are organized into big ideas, course competencies/standards, and learning objectives/expectations. The grade level key competencies represent the standard expectation of learning for students in each grade level. The competencies/standards are each followed by more detailed learning objectives that further explain the expectations for learning in the specific grade levels.

New instructional supports are included, such as clarification of language and expectations, as well as detailed examples. These have been provided for teaching professionals and stakeholders through the Evidence of Student Learning Column that accompanies each learning objective.

## Course Description:

The College Readiness Mathematics Capstone Course (CRM) is a fourth course option for students who have completed Advanced Algebra (or the equivalent). The course is designed to serve as a bridge for high school students who will enroll in non-STEM post-secondary study and will serve to meet the high school fourth course graduation requirement. The course has been approved by the University System of Georgia as a fourth mathematics course beyond Advanced Algebra (or the equivalent) for non-STEM majors, so the course will meet the needs of collegebound seniors who will not pursue STEM fields.

The focus of this course is on key content and practice standards to ensure that students will be ready for post-secondary academic courses and career preparation in non-STEM fields. The course will revisit and expand the understanding of content standards introduced in earlier mathematics courses and will emphasize numeracy, algebra and functions, geometry, and statistics in a variety of contexts. Instruction and assessment should include the appropriate use of manipulatives and technology. Mathematics concepts should be represented in multiple ways, such as concrete/pictorial, verbal/written, numeric/data-based, graphical, and symbolic. Concepts should be introduced and used, where appropriate, in the context of realistic experiences. The Standards for Mathematical Practice will provide the foundation for instruction and assessment. The content standards selected are essential for post-secondary preparation in non-STEM study.

Students will be expected to complete a mandatory capstone project where they select one of the areas listed in the standard to identify a problem and use mathematical modeling to address it.

## Prerequisite:

This course is designed for students who have successfully completed *Advanced Algebra / Algebra II (or the equivalent)*.

**Georgia’s K-12 Mathematics Standards - 2021  
Mathematics Big Ideas and Learning Progressions, High  
School**

## **Mathematics Big Ideas, HS**

<b>HIGH SCHOOL</b>
<b>MATHEMATICAL PRACTICES (MP)</b>
<b>MATHEMATICAL MODELING (MM)</b>
<b>NUMERICAL (QUANTITATIVE) REASONING (NR)</b>
<b>PATTERNING &amp; ALGEBRAIC REASONING (PAR)</b>
<b>FUNCTIONAL &amp; GRAPHICAL REASONING (FGR)</b>
<b>GEOMETRIC &amp; SPATIAL REASONING (GSR)</b>
<b>DATA &amp; STATISTICAL REASONING (DSR)</b>
<b>PROBABILISTIC REASONING (PR)</b>

The 8 Mathematical Practices and the Mathematical Modeling Framework are essential to the implementation of the content standards presented in this course. More details related to these concepts can be found in the links below and in the first two standards presented in this course:

[Mathematical Practices](#)

[Mathematical Modeling Framework](#)

# College Readiness Mathematics

## (Mathematics Capstone Course)

The seven course standards listed below are the key content competencies students will be expected to master in this course. Additional clarity and details are provided through the classroom-level learning objectives and evidence of student learning details for each course standard found on subsequent pages of this document.

### **COURSE STANDARDS**

***CRM.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration, and expression. Seek help and apply feedback. Set and monitor goals.***

***CRM.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.***

***CRM.NR.2: Utilize exact and approximate calculations to quantify real-world phenomena and solve problems.***

***CRM.PAR.3: Construct expressions, equations, and inequalities, and use them to represent and solve problems by choosing appropriate procedures and interpreting solutions in context.***

***CRM.FGR.4: Define, build and interpret functions that arise in various contexts by applying knowledge of the characteristics of the different families of functions, and analyze the effects of parameters.***

***CRM.GSR.5: Reason deductively and inductively about figures and their properties and make sense of geometric situations using measurements in real-world contexts.***

***CRM.DSR.6: Make sense of and reason about variation in data using graphs, tables and probability models to solve problems and draw appropriate conclusions from solutions.***

# College Readiness Mathematics (Mathematics Capstone Course)

<b>MATHEMATICAL MODELING</b>		
<b>CRM.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.</b>		
<b>Expectations</b>		<b>Evidence of Student Learning</b> (not all inclusive; see Grade Level Overview for more details)
CRM.MM.1.1	Explain contextual, mathematical problems using a mathematical model.	<b>Fundamentals</b> <ul style="list-style-type: none"> <li>• Students should be provided with opportunities to learn mathematics in the context of real-life problems.</li> <li>• Contextual, mathematical problems are mathematical problems presented in context where the context makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (model with mathematics).</li> </ul>
CRM.MM.1.2	Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.	<b>Relevance and Application</b> <ul style="list-style-type: none"> <li>• Students are expected to complete a mandatory capstone project where they select one of the areas listed in the standard to identify a problem and use mathematical modeling to address it.</li> <li>• Identification of fields of interest should occur early in the year, and teachers should incorporate real-world examples from the selected areas throughout the course.</li> <li>• The capstone project should be student-driven and based on the individual interests of each student.</li> <li>• Project guidelines and scoring guides will need to be provided.</li> </ul>
CRM.MM.1.3	Using abstract and quantitative reasoning, make decisions about information and data from a contextual situation.	
CRM.MM.1.4	Use various mathematical representations and structures with this information to represent and solve real-life problems.	

<b>NUMERICAL &amp; QUANTITATIVE REASONING – Real Number System</b>			
<b>CRM.NR.2: Utilize exact and approximate calculations to quantify real-world phenomena and solve problems.</b>			
<b>Expectations</b>		<b>Evidence of Student Learning</b> (not all inclusive; see Grade Level Overview for more details)	
CRM.NR.2.1	Through multi-step/multi-operational problems, perform mathematical operations on real numbers demonstrating fluency using the order of operations.		
CRM.NR.2.2	Represent and solve problems using proportional reasoning with ratios, rates, proportions, and scaling.	<b>Relevance and Application</b> <ul style="list-style-type: none"> <li>Relate rate reasoning to dimensional analysis and know why and how the process works and when to use it.</li> </ul>	<b>Example</b> <ul style="list-style-type: none"> <li>Students perform conversions problems related to the medical field.</li> </ul>
CRM.NR.2.3	Apply the rules of exponents to simplify numerical expressions, extending the properties of exponents to rational exponents.		
CRM.NR.2.4	Perform mathematical operations on real numbers to include numerical radical expressions and complex fractions.		
CRM.NR.2.5	Estimate solutions to problems with real numbers and use the estimates to assess the reasonableness of results in the context of the problem.	<b>Strategies and Methods</b> <ul style="list-style-type: none"> <li>This learning objective should be embedded throughout the course.</li> </ul>	

<b>PATTERNING &amp; ALGEBRAIC REASONING – Expressions, Equations, &amp; Inequalities</b>			
<b>CRM.PAR.3: Construct expressions, equations, and inequalities, and use them to represent and solve problems by choosing appropriate procedures and interpreting solutions in context.</b>			
<b>Expectations</b>		<b>Evidence of Student Learning</b> (not all inclusive; see Grade Level Overview for more details)	
CRM.PAR.3.1	Create equations in one variable and use them to solve problems.	<b>Fundamentals</b> <ul style="list-style-type: none"> <li>Include linear and quadratic equations.</li> <li>Include basic equations involving radical, rational, absolute value, exponential, and logarithmic equations.</li> </ul>	
CRM.PAR.3.2	Create inequalities in one variable and use them to solve problems.	<b>Fundamentals</b> <ul style="list-style-type: none"> <li>Include linear and absolute value inequalities.</li> </ul>	
CRM.PAR.3.3	Using multiple representations, solve equations and inequalities and use the solutions to draw reasonable conclusions about a situation being modeled, including possible constraints.	<b>Strategies and Methods</b> <ul style="list-style-type: none"> <li>Representations include tables, graphs, equations, charts, written explanations, etc.</li> </ul>	
CRM.PAR.3.4	Solve quadratic equations using a variety of methods.	<b>Strategies and Methods</b>	

		<ul style="list-style-type: none"> <li>Methods include taking the square root, factoring, completing the square, the quadratic formula, and technology.</li> </ul>
CRM.PAR.3.5	Rearrange literal equations to highlight a specified variable using the same reasoning as in solving equations.	
CRM.PAR.3.6	Solve inequalities in one variable graphically and algebraically.	<b>Fundamentals</b> <ul style="list-style-type: none"> <li>Algebraic solutions include inequalities, interval notation, and set notation.</li> </ul>
CRM.PAR.3.7	Using multiple methods, create and solve systems of linear equations and inequalities.	<b>Strategies and Methods</b> <ul style="list-style-type: none"> <li>Solve graphically, algebraically (elimination and substitution), and by using technology.</li> </ul>
CRM.PAR.3.8	Solve a simple system of equations consisting of a linear and a quadratic equation in two variables. algebraically and graphically.	<b>Strategies and Methods</b> <ul style="list-style-type: none"> <li>Solve graphically and algebraically (elimination and substitution).</li> </ul>

### **FUNCTIONAL & GRAPHICAL REASONING – Building & Interpreting Functions**

**CRM.FGR.4: Define, build and interpret functions that arise in various contexts by applying knowledge of the characteristics of the different families of functions, and analyze the effects of parameters.**

<b>Expectations</b>		<b>Evidence of Student Learning</b> (not all inclusive; see Grade Level Overview for more details)	
CRM.FGR.4.1	Define a function through maps, sets, equations and graphs using function notation.		
CRM.FGR.4.2	Identify and sketch by hand the parent graph of functions expressed algebraically and show key characteristics of the graph using technology.	<b>Strategies and Methods</b> <ul style="list-style-type: none"> <li>Identify characteristics including intercepts, domain, range, interval where the function is increasing, decreasing, positive, or negative, relative maximums and minimums, symmetries, and end behavior.</li> </ul>	<b>Fundamentals</b> <ul style="list-style-type: none"> <li>Include linear, quadratic, cubic, rational, square root, absolute value, exponential, and logarithmic functions.</li> </ul>
CRM.FGR.4.3	Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function.	<b>Strategies and Methods</b> <ul style="list-style-type: none"> <li>Interpret characteristics including intercepts, domain, range, interval where the function is increasing, decreasing, positive, or negative, relative maximums and minimums, symmetries, and end behavior.</li> </ul>	<b>Fundamentals</b> <ul style="list-style-type: none"> <li>Include linear, quadratic, cubic, rational, square root, absolute value, exponential, and logarithmic functions.</li> </ul>
CRM.FGR.4.4	Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph.		
CRM.FGR.4.5	Compare characteristics of two functions each represented in a different way.	<b>Strategies and Methods</b> <ul style="list-style-type: none"> <li>Include functions represented algebraically, graphically, numerically, or by verbal descriptions.</li> </ul>	

CRM.FGR.4.6	Construct linear and exponential functions, given a graph, a description of a relationship, or two input-output pairs.	<b>Strategies and Methods</b> <ul style="list-style-type: none"> <li>• Include reading input-output pairs from a table.</li> </ul>
CRM.FGR.4.7	Construct arithmetic and geometric sequences recursively and explicitly, use them to model situations, and translate between the two forms. Connect linear functions to arithmetic sequences and exponential functions to geometric sequences.	
CRM.FGR.4.8	Identify the effect on the parent graph of replacing $f(x)$ by $f(x) + k$ , $kf(x)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.	

**GEOMETRIC & SPATIAL REASONING – Measurement, Surface Area, Volume, & Right Triangle Trigonometry**

**CRM.GSR.5: Reason deductively and inductively about figures and their properties and make sense of geometric situations using measurements in real-world contexts.**

<b>Expectations</b>		<b>Evidence of Student Learning</b> (not all inclusive; see Grade Level Overview for more details)	
CRM.GSR.5.1	Use the distance formula, midpoint formula or slope to verify simple geometric properties.		
CRM.GSR.5.2	Use coordinates to compute perimeters of polygons, circumference of circles and areas of triangles, rectangles and circles.		
CRM.GSR.5.3	Informally derive the formulas for the volume and surface area of a cylinder, sphere, prism, pyramid, and cone.	<b>Strategies and Methods</b> <ul style="list-style-type: none"> <li>• Nets</li> </ul>	<b>Example</b> <ul style="list-style-type: none"> <li>• Students will use the peel of an orange to informally derive the surface area of a sphere.</li> </ul>
CRM.GSR.5.4	Use formulas for finding the volume and surface area of spheres, right and oblique prisms, cylinders, pyramids, and cones.		
CRM.GSR.5.5	Apply the Pythagorean Theorem and trigonometric ratios to solve problems involving right triangles.		

**DATA & STATISTICAL REASONING – Interpreting Data & Calculating Probabilities of Compound Events**

**CRM.DSR.6: Make sense of and reason about variation in data using graphs, tables and probability models to solve problems and draw appropriate conclusions from solutions.**

Expectations		Evidence of Student Learning (not all inclusive; see Grade Level Overview for more details)
CRM.DSR.6.1	Represent univariate data on the real number line.	<b>Strategies and Methods</b> <ul style="list-style-type: none"> <li>• Include dot plots, histograms, and box plots.</li> </ul>
CRM.DSR.6.2	Calculate, compare, and interpret shape, center, and spread of two or more univariate data sets, accounting for possible effects of extreme data points.	<b>Strategies and Methods</b> <ul style="list-style-type: none"> <li>• Measures of center include median and mean.</li> <li>• Measures of spread include interquartile range and standard deviation.</li> </ul>
CRM.DSR.6.3	Summarize categorical data for two categories in two-way frequency tables using relative frequencies in the context of the data.	<b>Strategies and Methods</b> <ul style="list-style-type: none"> <li>• Categorical data should include joint, marginal, and conditional relative frequencies.</li> </ul>
CRM.DSR.6.4	Represent bivariate data on a scatter plot and describe how the variables are related in terms of strength and direction.	
CRM.DSR.6.5	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	
CRM.DSR.6.6	Compute using technology and interpret the correlation coefficient “r” of a linear fit.	
CRM.DSR.6.7	Distinguish between correlation and causation, and interpolation and extrapolation.	
CRM.DSR.6.8	Describe categories of events as subsets of a sample space using unions, intersections, or complements of other events.	
CRM.DSR.6.9	Use the two-way frequency table to calculate conditional probabilities.	
CRM.DSR.6.10	Calculate the conditional probability of A given B.	

# **ESSENTIAL INSTRUCTIONAL GUIDANCE**

# MATHEMATICAL PRACTICES

The Standards for Mathematical Practices describe the reasoning behaviors students should develop as they build an understanding of mathematics – the “habits of mind” that help students become mathematical thinkers. There are eight standards, which apply to all grade levels and conceptual categories.

These mathematical practices describe how students should engage with the mathematics content for their grade level. Developing these habits of mind builds students’ capacity to become mathematical thinkers. These practices can be applied individually or together in mathematics lessons, and no particular order is required. In well-designed lessons, there are often two or more Standards for Mathematical Practice present.

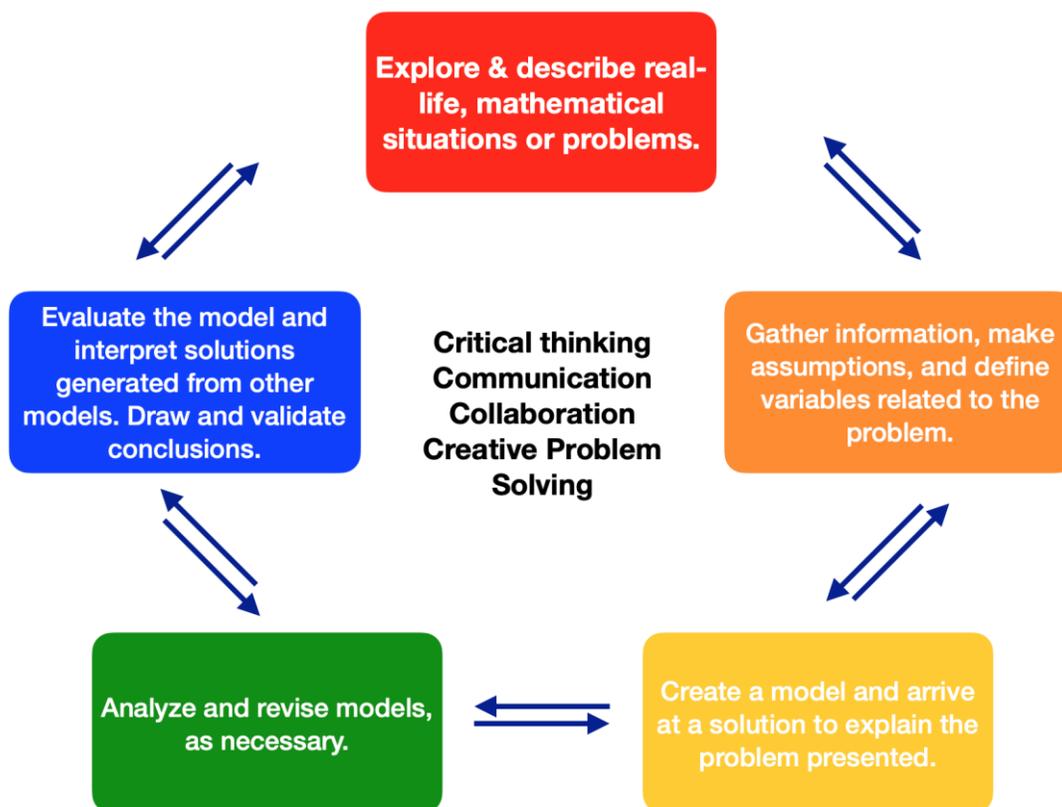
<b>Mathematical Practices</b>	
<b><i>CRM.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.</i></b>	
<b>Code</b>	<b>Expectation</b>
<b>CRM.MP.1</b>	Make sense of problems and persevere in solving them.
<b>CRM.MP.2</b>	Reason abstractly and quantitatively.
<b>CRM.MP.3</b>	Construct viable arguments and critique the reasoning of others.
<b>CRM.MP.4</b>	Model with mathematics.
<b>CRM.MP.5</b>	Use appropriate tools strategically.
<b>CRM.MP.6</b>	Attend to precision.
<b>CRM.MP.7</b>	Look for and make use of structure.
<b>CRM.MP.8</b>	Look for and express regularity in repeated reasoning.

# MATHEMATICAL MODELING

Teaching students to model with mathematics is engaging, builds confidence and competence, and gives students the opportunity to collaborate and make sense of the world around them, the main reason for doing mathematics. For these reasons, mathematical modeling should be incorporated at every level of a student's education. This is important not only to develop a deep understanding of mathematics itself, but more importantly to give students the tools they need to make sense of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed daily life decisions based on data and the models they create.

The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a real-life problem or task.

## A Mathematical Modeling Framework



*Image adapted from: Suh, Matson, Seshaiyer, 2017*

# FRAMEWORK FOR STATISTICAL REASONING

Statistical reasoning is important for learners to engage as citizens and professionals in a world that continues to change and evolve. Humans are naturally curious beings and statistics is a language that can be used to better answer questions about personal choices and/or make sense of naturally occurring phenomena. Statistics is a way to ask questions, explore, and make sense of the world around us.

The Framework for Statistical Reasoning should be used in all grade levels and courses to guide learners through the sense-making process, ultimately leading to the goal of statistical literacy in all grade levels and courses. Reasoning with statistics provides a context that necessitates the learning and application of a variety of mathematical concepts.

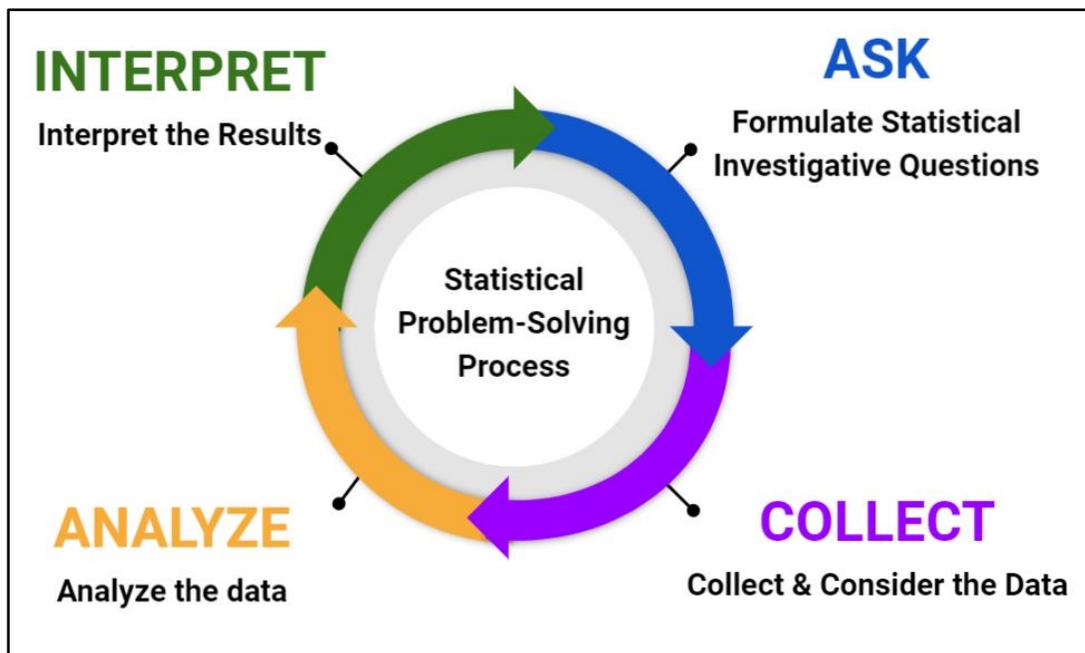


Figure 1: Georgia Framework for Statistical Reasoning

The following four-step statistical problem-solving process can be used throughout each grade level and course to help learners develop a solid foundation in statistical reasoning and literacy:

- I. Formulate Statistical Investigative Questions**  
Ask questions that anticipate variability.
- II. Collect & Consider the Data**  
Ensure that data collection designs acknowledge variability.
- III. Analyze the Data**  
Make sense of data and communicate what the data mean using pictures (graphs) and words. Give an accounting of variability, as appropriate.
- IV. Interpret the Results**  
Answer statistical investigative questions based on the collected data.